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RETSCP: A COMPUTER PROGRAM FOR ANALYSIS OF ROCKET ENGINE THERMAL STRAINS WITH CYCLIC PLASTICITY

by

Roy W. Miller

Atkins & Merrill Inc.

Ashland, Mass.

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H.G. Price, Project Manager

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SUMMARY

A computer program, designated RETSCP, for the analysis of Rocket Engine Thermal Strains with Cyclic Plasticity is described in detail. RETSCP is a finite element program which employs a three dimensional isoparametric element. The program treats elasto-plastic strain cycling including the effects of thermal and pressure loads and temperature dependent material properties. Theoretical aspects of the finite element method are discussed and the program logic is described. A RETSCP User's Manual is presented including sample case results.

INTRODUCTION

A new generation of high performance liquid rocket engines is being considered for Space Transportation System applications. The high performance goal for these engines demands high chamber pressures which result in high heat flux levels. Engine reusability is a prime objective. With the requirement of thermal and pressure cycling, the stress analyst must be able to define the life potential of a given design, considering cyclic fatigue where chamber wall stresses are sufficiently high to cause plastic strains.

The state of stress in regeneratively cooled rocket chambers varies in three dimensions. For such geometries, a numerical method of analysis must be employed. The numerical technique which has been given the most attention during the past decade is the finite element method. For an outstanding introduction to the finite element method, see Zienkiewicz's text, Reference 1.

The following report describes a finite element computer program designated RETSCP which was developed specifically for the purpose of Rocket Engine Thermal Strain analysis with Cyclic Plasticity. The program is an outgrowth of a General Electric program called ISOPAR, Reference 2.

ISOPAR employs a three-dimensional isoparametric element to compute the elastic stress distribution in structures which can be modeled with relatively few elements.

The transformation of ISOPAR into RETSCP followed a step-by-step approach. First, the program was expanded to allow for more elements in the structural model. Then, the capability of including thermal loads and computing thermal stresses was added. The program was next modified to allow non-zero prescribed displacements and to treat sliding boundaries. The symmetry condition in a rocket chamber is represented by a sliding boundary. Finally, plastic behavior with temperature dependent material properties was included. In conjunction with this final step, residual strains are output on punch cards to allow strain cycle restarts.

This report begins with a discussion of the theoretical aspects of the finite element method. The RETSCP program logic and computational scheme are then described. Finally, a RETSCP program User's Manual is given which includes sample case results. It is intended that a prospective program user can go directly to the User's Manual to obtain a working knowledge of the program. For application of the RETSCP program to specific rocket chamber analyses, see Reference 10.

FINITE ELEMENT METHOD

The theory of the finite element method has been well documented in several texts (c.f., Reference 1). There are many types of elements which have been developed, Reference 3. The choice between elements is this: use many simple elements, or use few complex elements. The isoparametric element, Reference 4, is a very complex element which leads to accurate results with a course structural model.

In this section, the theory of the finite element method is described with specific reference to the isoparametric element which is used in the RETSCP program. The stress-strain analysis, application of boundary conditions, thermal loading, and bi-linear plasticity models are discussed in the context of the RETSCP program.

General Theory

The finite element method is a procedure for approximating a continuum by an assembly of distinct elements having a finite number of unknowns. For structural analysis, this amounts to solving the force-displacement equations for the element assembly subject to the prescribed boundary values. That is, the following system of equations is formulated and solved:

$$\{F\} = [K]\{\delta\} \tag{1}$$

where, F and δ are the forces and displacements at the nodal points which connect the elements, and $[K]$ is the master stiffness matrix for the assembly. All symbols are defined in Appendix A. The appropriate force and displacement boundary conditions are used to obtain the solution to equation (1).

The master stiffness matrix is formed by assembling the individual stiffness matrices for each element. The element stiffness $[k]$ is determined by employing strain energy considerations. Apropos to these remarks, the strain within each element is related to the element nodal point displacements as follows:

$$\{\varepsilon\} = [B]\{\delta\} \tag{2}$$

For an elastic structure, the general stress-strain relationship is

$$\{\sigma\} = [D]\{\varepsilon\} \qquad (3)$$

Now, the aforementioned energy considerations (c.f. Reference 1) imply the following:

$$[k] = \int_{\text{volume}} [B]^T [D] [B] dV \qquad (4)$$

The functional relationship in equation (2) depends on the particular element employed. The detailed manner in which the integration, equation (4), is carried out also depends on the choice of element. The general procedure, however, is to solve the force-displacement equations for the assembly under the imposed boundary conditions.

<u>Isoparametric Element</u>

Following Reference 1, consider the eight node box element shown in Figure 1. The nodal points are located in space by their x-y-z coordinates in the rectangular right hand system. We introduce a set of parameters (ξ, η, ζ) such that their values are either +1 or -1 on the element faces. A set of eight linear functions of the parameters is then defined such that their functional value is +1 at each corresponding node and zero elsewhere.

That is,

$$N_1 = (1/8)\ (1-\xi)\ (1-\eta)\ (1-\zeta)$$

$$N_2 = (1/8)\ (1-\xi)\ (1+\eta)\ (1-\zeta)$$

$$N_3 = (1/8)\ (1+\xi)\ (1+\eta)\ (1-\zeta)$$

$$N_4 = (1/8)\ (1+\xi)\ (1-\eta)\ (1-\zeta)$$

$$N_5 = (1/8)\ (1-\xi)\ (1-\eta)\ (1+\zeta)$$

$$N_6 = (1/8)\ (1-\xi)\ (1+\eta)\ (1+\zeta)$$

$$N_7 = (1/8)\ (1+\xi)\ (1+\eta)\ (1+\zeta)$$

$$N_8 = (1/8)\ (1+\xi)\ (1-\eta)\ (1+\zeta) \qquad (5)$$

Note that these functions apply when the node numbering is such that nodes 1-2-3-4 go clockwise around the bottom when viewed from the top and nodes 5-6-7-8 are above nodes 1-2-3-4 respectively.

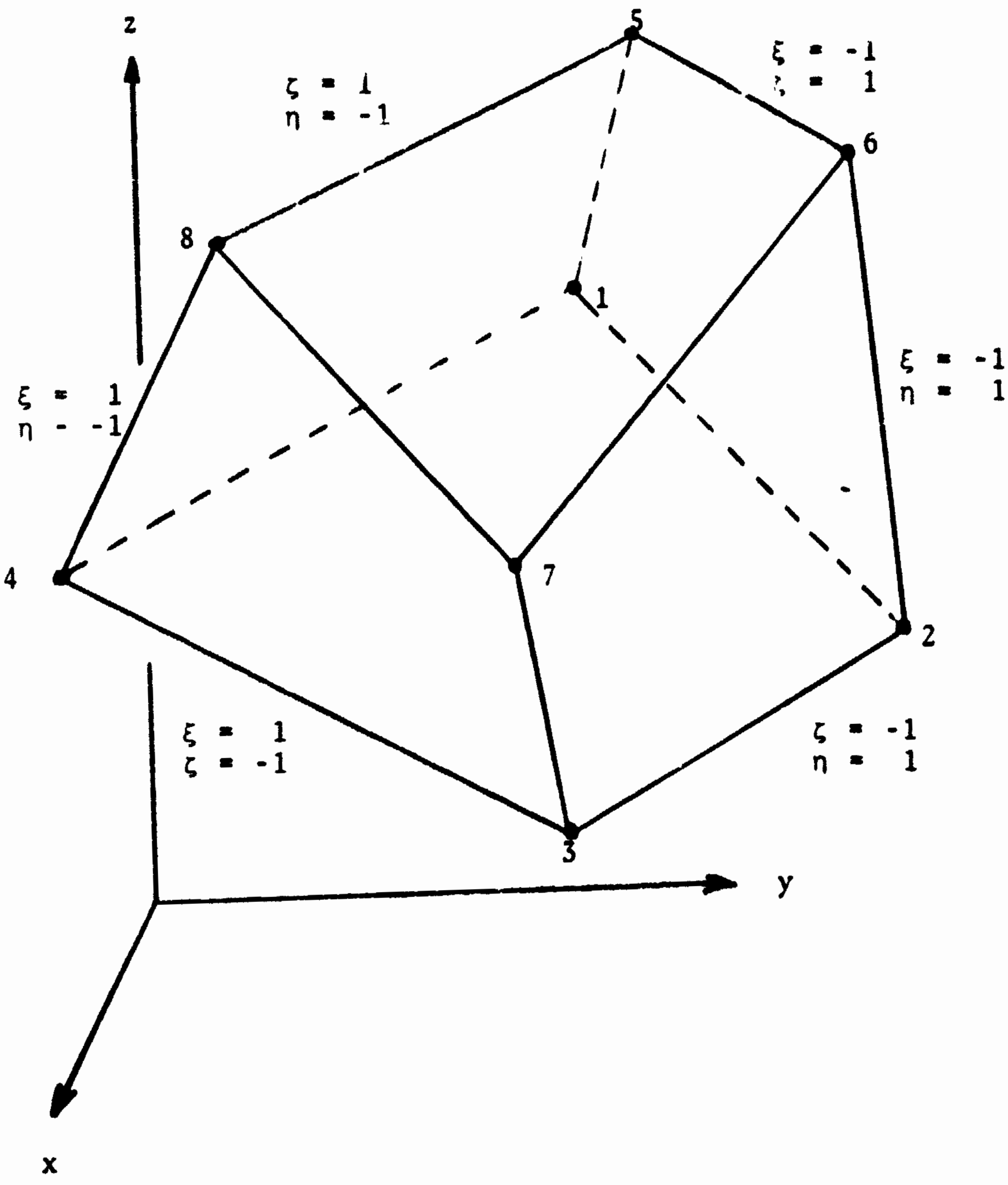


Figure 1. Rectangular and parametric coordinate systems for eight node box element.

Now, the coordinates of any point within the element x, y, z can be related to the coordinates of the eight nodal points x_n, y_n, z_n by the following parametric expressions:

$$x = N_1 x_1 + N_2 x_2 + \ldots N_8 x_8 = \{N_n\}^T\{x_n\}$$

$$y = N_1 y_1 + N_2 y_2 + \ldots N_8 y_8 = \{N_n\}^T\{y_n\}$$

$$z = N_1 z_1 + N_2 z_2 + \ldots N_8 z_8 = \{N_n\}^T\{z_n\} \qquad (6)$$

Equations (6) thus imply a relationship between (x, y, z) and (ξ, η, ζ).

Bear in mind, that our objective is to evaluate the stiffness matrix for the three-dimensional box element, equation (4). Thus, we require detailed expressions for the B-ma[illegible] and D-matrix. The stress matrix, D-matrix, for [illegible]otropic material with elastic modulus E, and Poisson's ra[illegible]o ν is:

$$D = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \nu/(1-\nu) & \nu/(1-\nu) & 0 & 0 & 0 \\ \nu/(1-\nu) & 1 & \nu/(1-\nu) & 0 & 0 & 0 \\ \nu/(1-\nu) & \nu/(1-\nu) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \qquad (7)$$

The B-matrix relates strain at any point in the element to the nodal point displacements. The general strain-displacement equations are:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial w/\partial z \\ \partial u/\partial y + \partial v/\partial x \\ \partial v/\partial z + \partial w/\partial y \\ \partial w/\partial x + \partial u/\partial z \end{Bmatrix} \qquad (8)$$

We relate the displacements of a point in space u, v, w to the nodal point displacements $\{u_n\}$, $\{v_n\}$, $\{w_n\}$ as follows:

$$u = N_1u_1 + N_2u_2 + \ldots N_8u_8 = \{N_n\}^T\{u_n\}$$

$$v = N_1v_1 + N_2v_2 + \ldots N_8v_8 = \{N_n\}^T\{v_n\}$$

$$w = N_1w_1 + N_2w_2 + \ldots N_8w_8 = \{N_n\}^T\{w_n\} \qquad (9)$$

An element, such as this, for which the same shape function expresses the element geometry and displacement fields is called an isoparametric element.

Substitution of equations (9) into equation (8) gives,

$$
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}
=
\begin{bmatrix}
\frac{\partial N_1}{\partial x} & 0 & 0 & \frac{\partial N_2}{\partial x} & 0 & 0 & \cdots & \frac{\partial N_8}{\partial x} & 0 & 0 \\
0 & \frac{\partial N_1}{\partial y} & 0 & 0 & \frac{\partial N_2}{\partial y} & 0 & \cdots & 0 & \frac{\partial N_8}{\partial y} & 0 \\
0 & 0 & \frac{\partial N_1}{\partial z} & 0 & 0 & \frac{\partial N_2}{\partial z} & \cdots & 0 & 0 & \frac{\partial N_8}{\partial z} \\
\frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial y} & \cdots & \cdots & \cdots & \frac{\partial N_8}{\partial y} & \frac{\partial N_8}{\partial x} & 0 \\
0 & \frac{\partial N_1}{\partial z} & \frac{\partial N_1}{\partial y} & 0 & \cdots & \cdots & \cdots & 0 & \frac{\partial N_3}{\partial z} & \frac{\partial N_8}{\partial y} \\
\frac{\partial N_1}{\partial z} & 0 & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial z} & \cdots & \cdots & \cdots & \frac{\partial N_8}{\partial z} & 0 & \frac{\partial N_8}{\partial x}
\end{bmatrix}
\begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ \vdots \\ \vdots \\ \vdots \\ u_8 \\ v_8 \\ w_8 \end{Bmatrix}
\qquad (10)
$$

To evaluate the displacement derivatives in equation (10), we make use of the Jacobian matrix. That is,

$$
[J] = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\
\frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta}
\end{bmatrix}
\qquad (11)
$$

Substituting equations (6) into equation (11) gives,

$$[J] = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} \cdots \frac{\partial N_8}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} \cdots \frac{\partial N_8}{\partial \eta} \\ \frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} \cdots \frac{\partial N_8}{\partial \zeta} \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ x_8 & y_8 & z_8 \end{bmatrix} \qquad (12)$$

The derivatives in equation (12) are readily obtained by differentiating equations (5). This matrix applies for all elements and, thus, need only be evaluated once. Then, we can determine the Jacobian at any position once the nodal point coordinates have been specified.

It turns out that the derivatives with respect to the physical coordinates are related to the parametric coordinates as follows:

$$\begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} \cdots \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} \cdots \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} \cdots \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} \cdots \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} \cdots \\ \frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} \cdots \end{bmatrix} \qquad (13)$$

The above matrix defines the elements of the B-matrix in equation (10). Thus, upon inverting the Jacobian matrix, the B-matrix can be readily evaluated at any point in the element.

Again we restate that our objective is to obtain the stiffness matrix, equation (4). Toward this goal we will make use of the following relation between element volumes in both coordinate systems:

$$dV_{xyz} = |J| dV_{\xi\eta\zeta} \tag{14}$$

where $|J|$ is the determinant of the Jacobian matrix. Then, the appropriate form of equation (4) to be evaluated is

$$[k] = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [B]^T [D] [B] |J| \; d\xi d\eta d\zeta \tag{15}$$

Equation (15) is evaluated numerically in the RETSCP program. The method employed is wo point Gaussian integration based on the following quadrature formula:

$$\int_{-1}^{1} f(\bar{x})d\bar{x} = f(+0.57735027) + f(-0.57735027) \tag{16}$$

Of course, the integration is carried out over three variables to evaluate equation (15). Thus, the terms in the integrand must be evaluated at eight Gauss points within the eight node box.

One key point remains to be made about the isoparametric element used in RETSCP. The element described above was based on eight linear shape functions, equations (5). The RETSCP element uses those eight functions plus the quadratic functions listed below:

$$
\begin{aligned}
N_9 &= 1 - \xi^2 \\
N_{10} &= 1 - \eta^2 \\
N_{11} &= 1 - \zeta^2
\end{aligned}
\tag{17}
$$

Including these, the element has 33 degrees of freedom (11 functions times 3 dimensions). Thus, the quadratic terms imply a higher order element. The functions, equations (17), are not associated with any specific point in space. For this reason, they are termed nodeless variables. The nine internal variables are eliminated internally within the program by the technique described in Zienkiewicz, Reference 1. Physically this amounts to separately minimizing strain energy with respect to the variables which are independent of the surroundings (otherwise called static condensation, Reference 3).

Finally, the stiffness matrix is obtained for each isoparametric element by the above procedure. Then, the master stiffness matrix can be assembled for the entire structure.

Boundary Conditions

Once the master stiffness matrix has been assembled, the objective is to solve the governing equations subject to the appropriate boundary conditions. That is, to solve the system of equations (1), which are rewritten below:

$$
\begin{Bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ F_n \end{Bmatrix} = \begin{Bmatrix} k_{11} & k_{12} & \cdot & \cdot & \cdot & \cdot & k_{1n} \\ k_{21} & k_{22} & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & & \cdot & & & & \cdot \\ \cdot & & & \cdot & & & \cdot \\ \cdot & & & & \cdot & & \cdot \\ \cdot & & & & & \cdot & \cdot \\ k_{n1} & \cdot & \cdot & \cdot & \cdot & \cdot & k_{nn} \end{Bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \delta_n \end{Bmatrix} \qquad (18)
$$

The stress boundary condition is automatically satisfied. Namely, forces at nodes on a free-surface are zero in the normal direction.

Prescribed Boundary Forces: Prescribed force values of P_j at the corresponding node are treated simply by replacing F_j by P_j in the force vector.

Prescribed Displacements: Prescribed displacement conditions are treated by modifying the force vector and stiffness matrix. Say the jth displacement is to be prescribed as α_j. First, replace F_j by $\bar{F}_j$ where

$$\bar{F}_j = F_j - \alpha k_{ji} \qquad (19)$$

Then, replace the jth row and column in the stiffness matrix by zero except k_{jj} which is replaced by 1. This is tantamount to eliminating one equation; yet the size of the matrix is not reduced.

As an example of the above procedure, assume u_1 has the prescribed value α. Then, the resulting equations are

$$\begin{Bmatrix} \alpha \\ F_2 - \alpha k_{12} \\ F_3 - \alpha k_{13} \\ \cdot \\ \cdot \\ F_n - \alpha k_{1n} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & k_{22} & k_{23} & \cdot & \cdot & \cdot & k_{2n} \\ \cdot & & \cdot & & & & \\ \cdot & & & \cdot & & & \\ \cdot & & & & \cdot & & \\ 0 & k_{n2} & & & & & k_{nn} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{Bmatrix} \qquad (20)$$

Symmetry Condition: The symmetry condition is represented by zero displacement normal to the plane of symmetry and no restraint along the plane of symmetry (sliding boundary). The symmetry plane is often skew with respect to the physical coordinate axis. This is the case for a wedge segment with axi-symmetry. Thus, we will derive a transformation to treat skew boundary conditions.

Referring to Figure 2, the displacements in the (x, y) system are (u, v). The skew system (x', y') has a rotation of the x-axis of magnitude θ (positive for rotation of x-axis toward y-axis). The displacements are related as follows:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} u' \\ v' \end{Bmatrix} = \begin{bmatrix} L \end{bmatrix} \begin{Bmatrix} u' \\ v' \end{Bmatrix} \qquad (21)$$

The original element properties were evaluated in the unprimed system, namely,

$$\{F\} = [K]\{\delta\} \qquad (22)$$

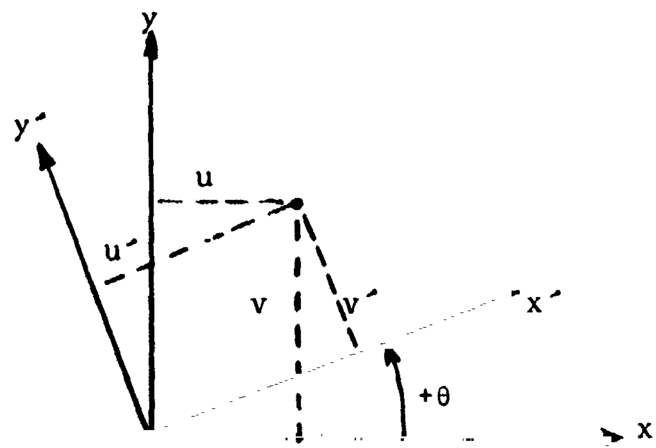


Figure 2. Notation for coordinate transformation.

The amount of work done is the same in both systems. That is,

$$\{F'\}^T\{\delta'\} = \{F\}^T\{\delta\} = \{F\}^T[L]\{\delta'\} \tag{23}$$

or

$$\{F'\} = [L]^T\{F\} = [L]^T[K][L]\{\delta'\} \tag{24}$$

Thus, we introduce the modified stiffness matrix below

$$[K'] = [L]^T[K][L] \tag{25}$$

If, the boundary conditions are introduced in skew coordinate directions; then, the corresponding force and displacement results are in the skew directions. The entire procedure is carried out internally within the program by multiplying

the appropriate rows and columns in the master stiffness matrix by the appropriate sin-cos terms. It goes without saying that only those nodes with skew coordinates need be treated. The final results are then transformed back into the physical coordinate systems.

Method of Solution

The set of governing equations is solved in the RETSCP program by Gaussian elimination. The master stiffness matrix is partitioned in the interest of computational efficiency. The governing equations can be written as matrix equations in terms of submatrices. For example,

$$\begin{bmatrix} \bar{K}_{11} & \bar{K}_{12} \\ \bar{K}_{21} & \bar{K}_{22} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \tag{26}$$

The term Δ_1 is eliminated from equation (26) to give:

$$[K^*]\{\Delta_2\} = \{F^*\} \tag{27}$$

where,

$$[K^*] = [K_{22}] - [\bar{K}_{21}][\bar{K}_{11}]^{-1}[\bar{K}_{12}] \tag{28}$$

$$\{F^*\} = \{F_2\} - [\bar{K}_{21}][\bar{K}_{11}]^{-1}\{F_1\} \tag{29}$$

Equation (27) can be solved to give $\{\Delta_2\}$ by premultiplying by the inverse matrix $[K^*]^{-1}$. Then, back substitution yields the following:

$$\{\Delta_1\} = [\bar{K}_{11}]^{-1}\{F_1\} - [\bar{K}_{11}]^{-1}[\bar{K}_{12}]\{\Delta_2\} \tag{30}$$

Alternately, equation (27) can be partitioned and the same procedure reapplied to further reduce the system.

It should also be noted that the master stiffness is a banded matrix. This fact also leads to a simplification in the matrix manipulation. Consider the following:

$$[K] = \begin{bmatrix} \bar{K}_{11} & \bar{K}_{12} & 0 \\ \bar{K}^T_{12} & \bar{K}_{22} & \bar{K}_{23} \\ 0 & \bar{K}^T_{32} & \bar{K}_{33} \end{bmatrix} \tag{31}$$

Elimination of $\bar{K}_{11}$ causes no change in $\bar{K}_{23}$ or $\bar{K}_{33}$. Thus, only $\bar{K}_{22}$ need be modified. (See Reference 1).

Thermal Strain Effects

The previous development was based on elastic deformation of an isothermal structure. In this section, the method of including thermal effects is described; also, see Reference 5.

The temperature difference, referred to a stress free state, is input data for each element. Of course, a suitable average value must be used for each entire element. The free thermal growth of each element is computed. Based on the element stiffness, the nodal forces required to mechanically produce the thermal growth are determined. These forces are then added to the force vector of the entire assembly. Loads and deflections are computed as usual for the assembled structure. The stress results are adjusted by adding the fully restrained thermal stress level for each element. The result is then the actual mechanical stress state.

<u>Bi-Linear Plasticity</u>

The RETSCP program treats plastic material behavior by adjusting the material properties and iterating upon the elastic solution. This is the secant modulus procedure which was employed in many previous two dimensional finite element programs (c.f., References 6 and 7).

A complete treatment of plastic material behavior is given in Reference 8. For the purpose at hand, it is sufficient to say that total deformation theory is used; and, yielding is based on the Von Mises criteria. For each element in

the structure, the average value of the equivalent (or effective) stress is computed. That is, the average value of the following:

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2+(\sigma_x-\sigma_z)^2+(\sigma_y-\sigma_z)^2+ 6(\tau^2_{xy}+\tau^2_{yz}+\tau^2_{xz})\right]^{\frac{1}{2}} \qquad (32)$$

Then, according to the Von Mises yield criteria, yielding occurs if σ_e is greater than the yield stress from the uniaxial stress-strain test. For plastic behavior, equivalent stress and plastic strain are related via the uniaxial stress-strain curve as shown in Figure 3.

The RETSCP program employs a bi-linear approximation for the uniaxial stress-strain curve. The curve is defined by elastic modulus E, yield stress level σ_y, and plastic modulus mE. Plastic modulus and yield can be input as functions of temperature. An example of the bi-linear stress-strain curve is shown on Figure 4.

The essence of the secant modulus formulation is as follows. First, conduct an elastic structural analysis. Compute effective stress and check each element for yielding. For elements which indicate yielding, define a new elastic modulus called the secant modulus. The secant modulus is based on the bi-linear stress-strain curve at the strain level corresponding to the elastic result; that is, ε_{total}.

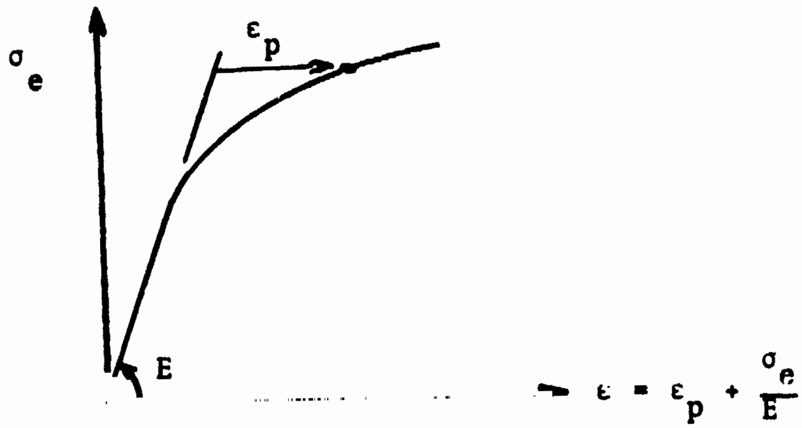


Figure 3. Relation between equivalent stress and equivalent plastic strain.

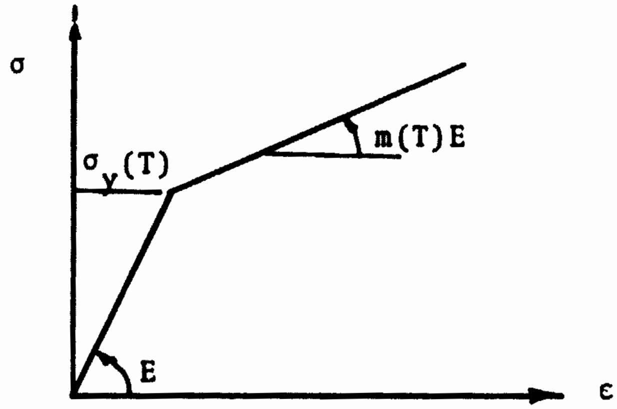


Figure 4. Bi-linear stress-strain curve.

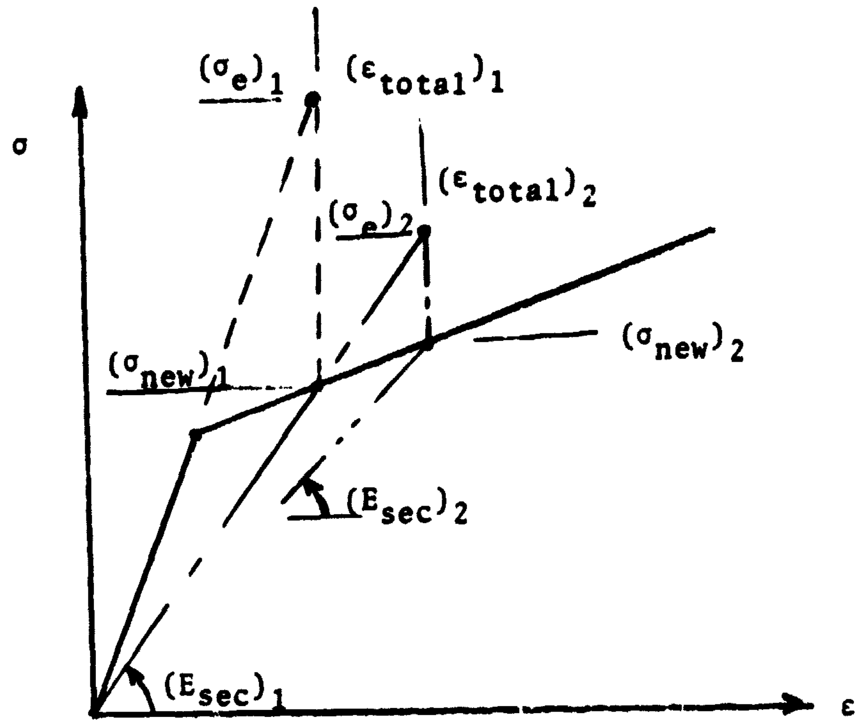


Figure 5. Secant modulus plasticity iteration.

Associated with ϵ_{total} is a bi-linear stress intercept σ_{new}. The secant modulus is defined below:

$$E_{sec} = \frac{\sigma_{new}}{\epsilon_{total}} \tag{33}$$

The secant Poisson's ratio, defined to give a consistent stress-strain relation, is as follows:

$$\nu_{sec} = \frac{1}{2} - \left(\frac{1}{2} - \nu\right)\frac{E_{sec}}{E} \tag{34}$$

Now, an elastic analysis is again conducted. The stiffness matrix, however, is based on E_{sec} and ν_{sec} for plastic elements and E and ν for elastic elements. The entire procedure is repeated and convergence is achieved after a few iterative cycles. The process is indicated schematically in Figure 5.

Cyclic Loading

Two effects of cyclic loading must be considered. First, there is the effect of cycling on the material properties (see Reference 9). The effect of strain hardening (or softening) can be introduced in the program on a cycle by cycle basis; or, the cyclic stress-strain curve can be input.

The second effect is the result of plastic deformations during one half of the loading cycle. Upon removal of the load, residual stresses (or strains) result when plastic flow has occured. The residuals, in fact, may be sufficiently large to also cause plastic deformation. Thus, a stress (or strain cycle) is generated.

The plastic strain components are related to the stress, effective stress, and effective plastic strain as follows:

$$\varepsilon_x^p = \frac{\varepsilon_p}{2\sigma_e}(2\sigma_x - \sigma_y - \sigma_z)$$

$$\varepsilon_y^p = \frac{\varepsilon_p}{2\sigma_e}(2\sigma_y - \sigma_x - \sigma_z)$$

$$\varepsilon_z^p = -(\varepsilon_x^p + \varepsilon_y^p)$$

$$\gamma_{xy}^p = \frac{3}{2}\frac{\varepsilon_p}{\sigma_e}\tau_{xy}$$

$$\gamma_{yz}^p = \frac{3}{2}\frac{\varepsilon_p}{\sigma_e}\tau_{yz}$$

$$\gamma_{xz}^p = \frac{3}{2}\frac{\varepsilon_p}{\sigma_e}\tau_{xz} \tag{35}$$

For rocket engine configurations, the shear strains are relatively small. Another quantity of interest is the equivalent total strain. This value is computed from the total strain components as follows:

$$\varepsilon_{et} = \frac{\sqrt{2}}{3}\left[(\varepsilon_x-\varepsilon_y)^2+(\varepsilon_x-\varepsilon_z)^2+(\varepsilon_y-\varepsilon_z)^2+6(\gamma_{xy}^2+\gamma_{xz}^2+\gamma_{yz}^2)\right]^{\frac{1}{2}} \tag{36}$$

The plastic strain based on this value is then

$$\varepsilon_p = \varepsilon_{et} - \frac{2}{3}\frac{1+\nu}{E}\sigma_e \tag{37}$$

Equivalent total strain, in itself, has no physical significance. Within the RETSCP program, the plastic strain components and equivalent total strain are computed for each element which has yielded. The residual strain components are then provided as punch card output for successive run calculations.

The residual strains are read into the program as input data for the computation of successive loadings. The strains are combined with the thermal strains and analyzed in the same manner. That is, the loads at each nodal point required to produce the residual strain values are computed and added to the assembled load vector. This point will be emphasized by example in a later section of this document.

PROGRAM LOGIC

The RETSCP program logic is described in this section. The general logic is discussed and the program flow iagram is given. Some specific points are made concerning the subroutine details. The detailed listing of the RETSCP program is given in Appendix B.

General Logic

The general RETSCP program logic is to follow the analytical procedures outlined in the previous chapter to obtain the desired finite element results.

The computational logic is controlled by the main program RETSCP. Subroutines are called as required to perform specific calculations. An overlay structure for subroutines is employed to reduce core storage requirements. In this manner, a specific calculation is performed in a subroutine, the results are put onto tape storage (seven tape units are utilized), and core storage locations occupied by that subroutine are released for reuse.

The above core storage management procedures allowed the RETSCP program size (number of elements) to be greatly enlarged from the original ISOPAR program size. In fact, the program was enlarged to fully utilize the available core storage of the IBM 7094 computer.

The data is read into RETSCP from punch cards. For each element, the elastic properties and stiffness matrix are computed (FEM3D). The master stiffness matrix is formed and the boundary values are incorporated (MATRIX). The system of equation is solved by Gaussian elimination (SOLVE), and the resulting force and displacement values at each nodal point are printed out. The elastic stress components and equivalent stress values are computed for each element (STRESS). Now, if the equivalent stress exceeds the yield stress a plastic iteration is performed. The iteration consists of: first, compute the values of secant modulus and Poisson's ratio (STRESS); then, use these values to recompute the elastic properties and stiffness matrix for each element (FEM3D); finally, complete the solution steps above. When the required number of iterations have been p rformed, the stress results are printed and the residual plastic strains and current secant modulus values are punched on cards to allow cycling and restart.

The flow diagram representing the above steps is given in the following section.

Flow Diagram

MAIN

ISOPAR → Compute isoparametric data at each Gauss point. (GP)

$$\begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \cdots & \frac{\partial N_{11}}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \cdots & \frac{\partial N_{11}}{\partial \eta} \\ \frac{\partial N_1}{\partial \zeta} & \cdots & \frac{\partial N_{11}}{\partial \zeta} \end{bmatrix}_{GP}$$

READIN → read in data and for each element call FEM3D ← **FEM3D**

$$D(6,6) = \frac{E(1-\nu)}{(1-2\nu)(1+\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$$

$$\varepsilon_0 = \alpha \Delta T$$

$$\delta_{thx} = \varepsilon_0 L_x$$

$$\sigma_0 = D\varepsilon_0$$

compute Equation (12) $[J]_P$

Equation (13)

$$\left[\frac{\partial N_1}{\partial x} \cdots \right] = [J]^{-1} \left[\frac{\partial N_1}{\partial \xi} \cdots \right]_{GP}$$

Set up B (6,33)

$\{\varepsilon\} = [B]\{\delta\}$

Equation (10)-modified

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_9}{\partial x} & 0 & 0 & \frac{\partial N_{10}}{\partial x} & \cdots \\ 0 & \frac{\partial N_9}{\partial y} & 0 & 0 & \cdots \\ 0 & 0 & \frac{\partial N_9}{\partial z} & 0 & \cdots \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \end{bmatrix} \begin{Bmatrix} u_9 \\ v_9 \\ w_9 \\ \vdots \\ w_{11} \\ u_1 \\ \vdots \\ w_8 \end{Bmatrix}$$

Set up stress matrix for each GP

A (6,33) = DB

Eliminate internal nodes

$(C7)_{GP}$ = C7 (6,24)

Compute stiffness matrix

C(24,24) = $[k]_{GP}$

$F_{th} = C\delta_{th}$

READIN

FACE

Use linear interpolation to get stress matrix at center of each face from those at GP, DBA,(6,24) each face.

MATRIX

Set up stiffness matrix for each partition

$$\begin{bmatrix} ST & ST & & \\ & ST & ST & \\ & & & \end{bmatrix}$$

Set up load vector

$F = F + F_{th}$

Insert boundary values
Equation (20)

$$\begin{Bmatrix} 0 \\ F_2 - k\delta_1 \\ F_3 - k\delta_2 \\ \\ \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \\ 0 & ST & & \\ 0 & & \cdot & \\ & & & \cdot \\ & & & & \cdot \end{bmatrix} \begin{Bmatrix} \delta \end{Bmatrix}$$

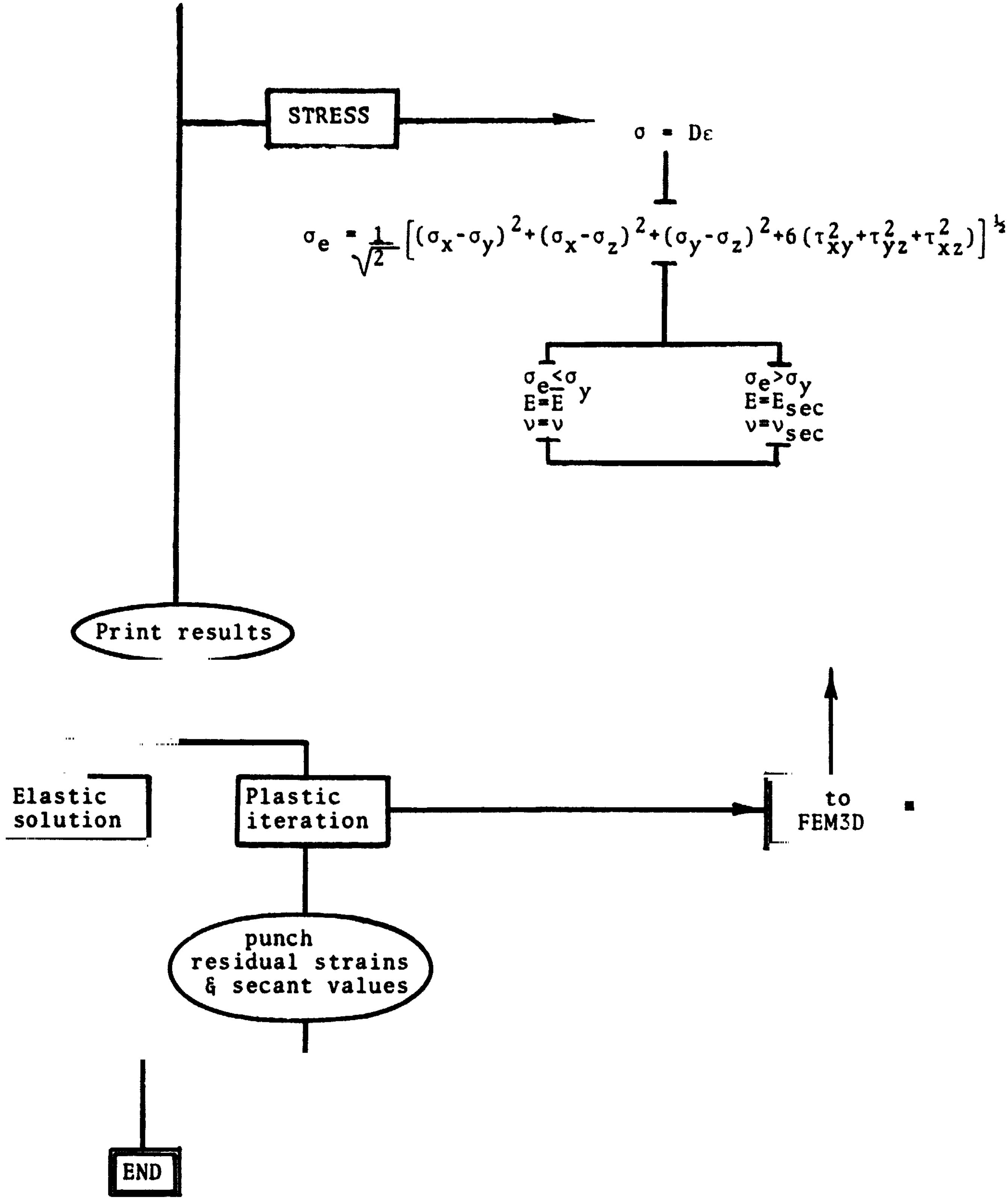
SOLVE

Gaussian elimination--
Equation (26)

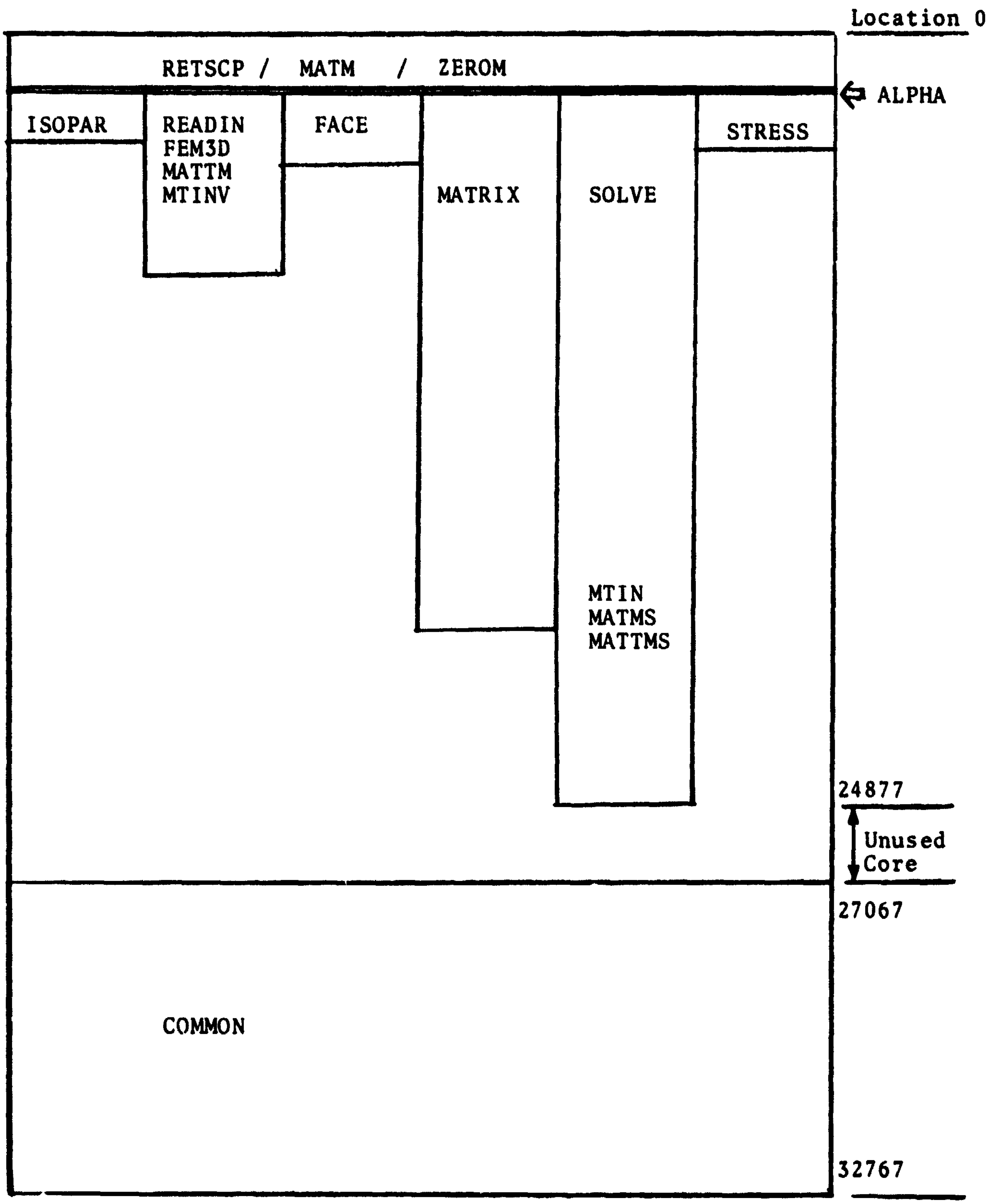
$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} \\ \bar{k}_{21} & \bar{k}_{22} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix}$$

result

$$\begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ \vdots \\ w_{last} \end{Bmatrix}$$


STRESS
$\sigma = D\varepsilon$
$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2+(\sigma_x-\sigma_z)^2+(\sigma_y-\sigma_z)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2)\right]^{\frac{1}{2}}$
$\sigma_e \leq \sigma_y$
$E = E$
$\nu = \nu$
$\sigma_e > \sigma_y$
$E = E_{sec}$
$\nu = \nu_{sec}$
Print results
Elastic solution
Plastic iteration
to FEM3D
punch residual strains & secant values
END

Overlay Structure



USER'S MANUAL

The User's Manual section contains all instructions necessary to prepare data for the RETSCP program. Modeling of the structure and preparation of the required input data cards are described in detail. Some comments about program output are included and sample case results are given.

Input

The input for RETSCP consists of punch card data which defines the structural geometry, boundary conditions, and materials properties.

The structure is divided into box shaped elements which are connected by corner nodes. The following procedure for locating nodes and elements is quoted directly from Reference 2.

> (a) The 3-dimensional solid is divided by a number of non-intersecting surfaces. (Much like slicing a loaf of bread.) The surfaces need not be flat or parallel, though they frequently are.
>
> (b) Each such surface is further subdivided by a number of non-intersecting lines. (Much like the lines on a piece of paper.) The lines need not be straight or parallel, though they frequently are.

(c) Each such line is further subdivided into a number of divisions to give the nodal points. Nodal points are numbered in sequence along each line, line by line, and surface by surface.

(d) The nodes on each surface are said to belong to the same partition. Partitions are numbered in sequence from one side of the solid to the other. (The first partition contains the first nodal points.)

(e) The number of divisions in adjacent lines can vary to provide for grading of the mesh.

(f) 8-noded box elements are formed between adjacent surfaces. They are numbered sequentially between each pair of adjacent surfaces. The numbering continues for successive adjacent surfaces in turn going from one side to the other of the solid structure. (Although in theory the boxes need not be "square", it is recommended that they be as "square" as the shape of the structure permits.) The first element has nodes in the first partition.

The detailed data cards required to execute the RETSCP program are listed below. Examples of the data preparation will be given in a subsequent section.

Card Group 1: Identification Card

Number of Cards: 1

Format: (11I4)

1. Number of partitions (9 maximum)
2. Number of nodes (225 maximum/25 per partition maximum)
3. Number of elements (96 maximum/32 per partition maximum)
4. Number of prescribed displacement nodes (225 maximum)
5. Number of materials (5 maximum)
6. Number of degrees of freedom at each node (always 3)
7. Number of nodes with applied loads (225 maximum)
8. Starting plasticity iteration number: 1, no iterations
   or 2, iteration starting from elastic solution
   or n, iteration starting from punch card input
   based on iteration number (n-1).
9. Final plasticity iteration number
10. Punch output code for successive iterations: 0, no punch output
    or 1, provide punch output
11. Residual stress code: 0, no residual strains input
    or 1, read residual strain card data

Card Group 2:	Coordinate Data	
	Number of Cards:	1 per node in order
	Format:	3F16.4

1. x-coordinate (inches)
2. y-coordinate (inches)
3. z-coordinate (inches)

Card Group 3:	Node Number Card	
	Number of Cards:	1
	Format:	I4

1. Number of nodes

Card Group 4:	Partition Identification	
	Number of Cards:	1 per partition in order
	Format:	4I4

1. First element number in partition
2. Last element number in partition
3. First node number in partition
4. Last node number in partition

Card Group 5:	Materials Identification		
	Number of Cards:	2 cards per material	
	Format:	first card	3F16.4
		second card	4F16.4

Card 1:
1. Young's modulus (psi)
2. Poisson's ratio
3. Coefficient of thermal expansion times 10^6(in/in/$^{\circ}$F)

Card 2:
1. Yield stress at reference temperature (psi), τ_0
2. Yield temperature gradient (psi/$^{\circ}$F), λ_1
3. Plastic modulus times 10^3 at reference temperature, m_0
4. Plastic modulus temperature gradient times 10^6(1/$^{\circ}$F), λ_2

Note, Card 2 values above are based on the following equations:

$$\sigma_y = \sigma_0 - \lambda_1 T \tag{38}$$

$$m = m_0 x 10^{-3} - \lambda_2 T x 10^{-6} \tag{39}$$

The value of T must correspond to the reference value on Card Group 7.

Card Group 6:	Prestrain Data (can be omitted)	
	Number of Cards:	1 per element
	Format:	I6, 3F15.8

1. Element Number
2. Prestrain in x-direction
3. Prestrain in y-direction
4. Prestrain in z-direction

Card Group 7:	Element Identification	
	Number of Cards:	1 per element in order
	Format:	9I4, F10.3

1.-8. Eight nodal point numbers

9. Material Number

10. Temperature excess over reference value

The eight nodal numbers referred to above are obtained for each element:

(a) Pick a face to be called the top;

(b) Look down through the top to the bottom face;

(c) List node numbers clockwise around the bottom face (4 values);

(d) List coincident node numbers clockwise around the top face (4 values) starting with the node above the first node on the bottom face.

Card Group 8.	Element Number Card
	Number of Cards: 1
	Format: 14
1. Number of elements	

Card Group 9:	Displacement Boundary Conditions
	Number of Cards: 1 for each node with prescribed displacement
	Format: 4I4, 4F16.8

1. Nodal number
2. 0 if x-displacement is prescribed; 1 if not
3. 0 if y-displacement is prescribed; 1 if not
4. 0 if z-displacement is prescribed; 1 if not
5. value of x-displacement (inches)
6. value of y-displacement (inches)
7. value of z-displacement (inches)
8. angle of rotation of x-axis toward original y-axis (deg.)

Sufficient displacement boundary condition data must be given to fix the body in space.

Card Group 10:	Force Boundary Conditions	
	Number of Cards:	1 per node with prescribed force
	Format:	I4, 4F16.4

1. Nodal number
2. x-force (pounds)
3. y-force (pounds)
4. z-force (pounds)

Card Group 11:	Iteration Data (can be omitted)	
	Number of Cards:	1 per element
	Format:	I6, F20.2, F10.4

1. Element number
2. Secant Young's modulus (psi)
3. Secant Poisson's ratio

Output

The RETSCP output consists of punched cards and printed data.

Punch cards are provided in conjunction with plastic strain analysis. If requested per Card Group 1, the secant modulus and secant Poisson's ratio are punched after the final iteration of that run. This allows the iterative process to be continued without recomputing the initial iterations. For plasticity analysis, the residual plastic strain values are automatically punched for the final iteration of that run. This data can be input directly for subsequent strain cycling calculations (Card Group 6). Secant values and residual strains are automatically printed at the end of the printed output when the above cards are punched.

The printed output starts with a list of the input data. Note, that the formats may be slightly different from the input. For example, Cards 1 and 2 in Group 5 are printed in reverse order (Card 2, then Card 1). Also Card Groups 3 and 8 are omitted. The input data is printed for checking and debug purposes.

The forces and displacements at each nodal point are listed. Values are given in the rotated and rectangular coordinate systems. The nodal force data output was incorporated to allow numerical evaluation of the net section force (such as rocket engine thrust force).

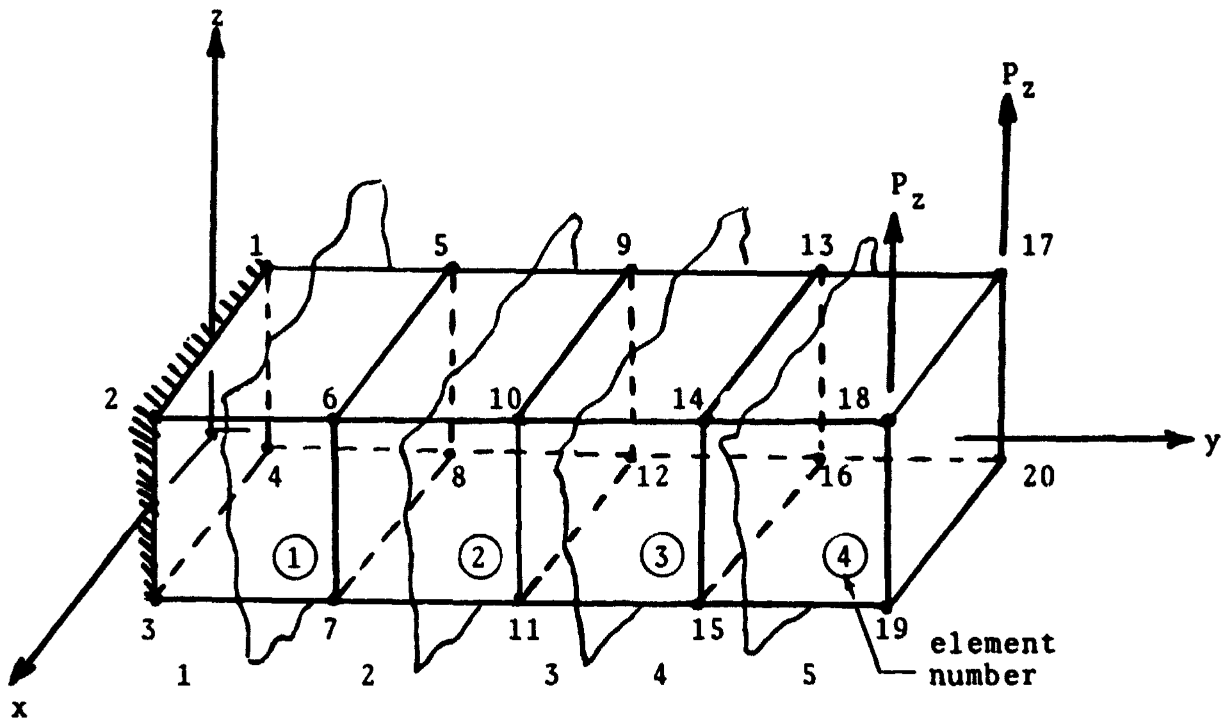
Detailed stress-strain data is given for each element. The stress and strain components at the center of each element face are printed as well as the coordinates of the face center point. The average stress components for each element are also listed. The effective stress which is computed in the program is based on the average stress components. The yield check data are then summarized in the output. This summary consists of effective stress, yield stress, total strain, plastic strain, and secant values for each element.

If plasticity iterations are performed; then all of the above output data is given for each iteration. Samples of output data will be presented as part of the next section.

Sample Case Results

Three sample case solutions are presented in this section. The cases were selected to demonstrate the capabilities of the RETSCP program by successively introducing new concepts. Elastic behavior of an isothermal structure is treated first. Then, sliding boundaries and plastic strains are introduced. Finally, thermal loads and strain cycling are illustrated.

Cantilever Beam: Consider the cantilever beam with concentrated tip load shown in Figure 6. The material is steel and the tip load is sufficiently low that elastic behavior is guaranteed. The beam is divided into four elements as shown in Figure 6. The input data and computer output results are presented in Appendix C. The bending stress at the outer fiber is compared with the exact solution in Figure 7. The deflection of the nodal points normal to the neutral axis (δ_z) is compared with the exact result in Figure 8. This example illustrates that excellent results can be achieved with models having few elements.



Load: P_z = 0.5 lbs. (2.224 Newton)

Size: L_x = 1.0 in. (2.54 cm)

L_y = 4.0 in. (10.16 cm)

L_z = 1.0 in. (2.54 cm)

Matl.: E = 30 x 10^6 psi (20.68 x 10^6 N/cm^2)

Figure 6. Cantilever beam sample case configuration

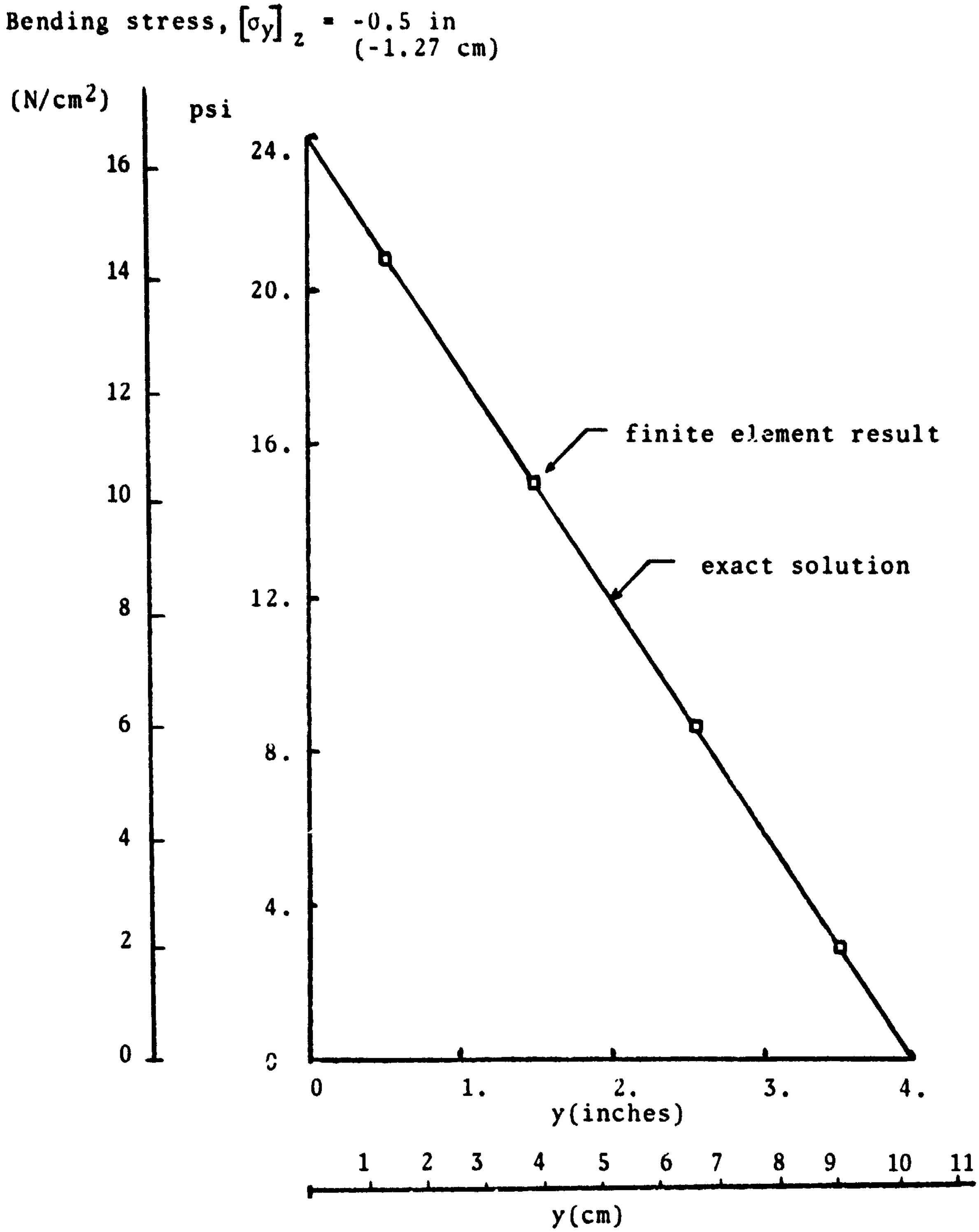


Figure 7. Outer fiber bending stress for cantilever beam example.

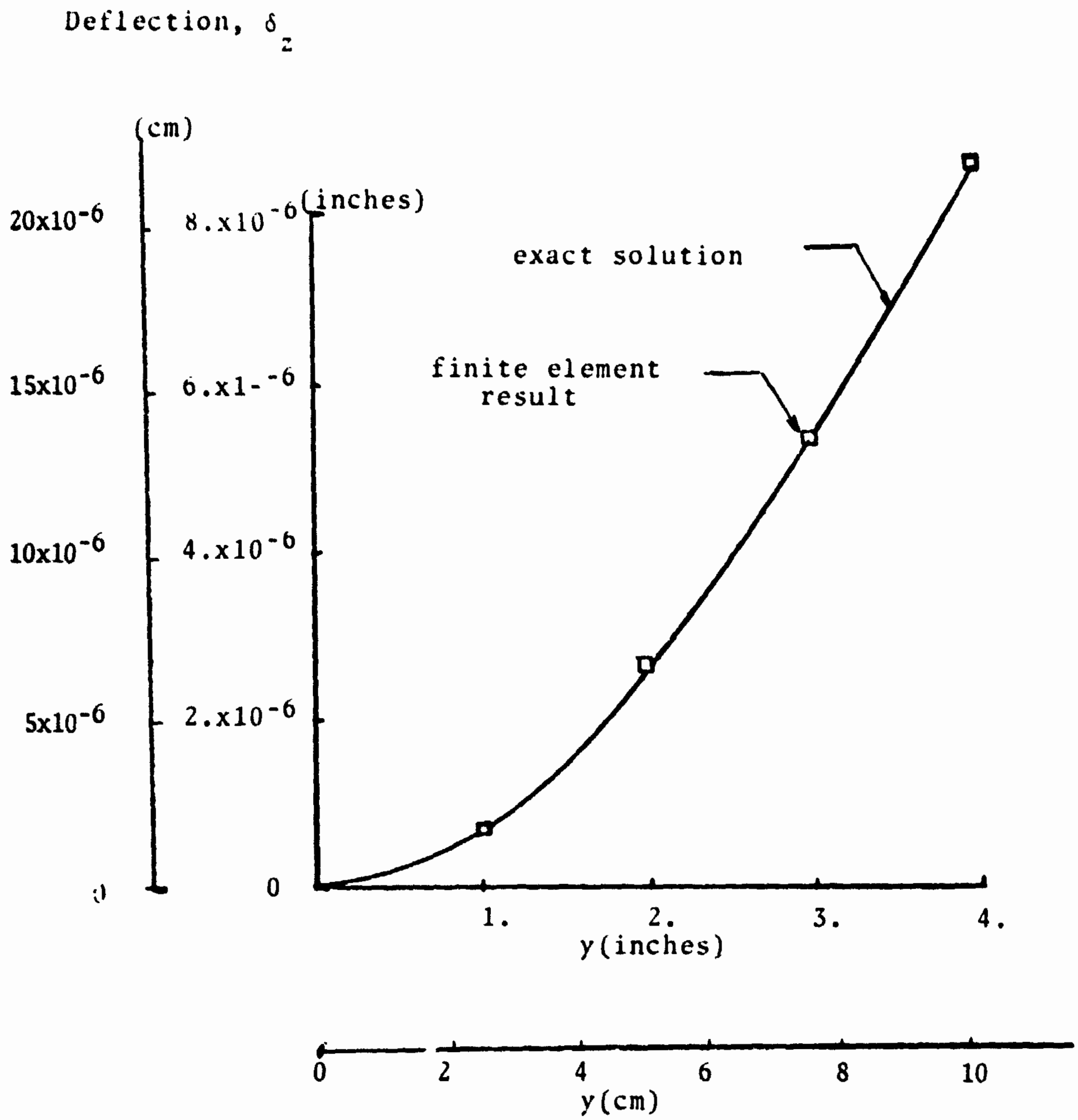


Figure 8. Nodal point deflection (δ_z) for cantilever beam example.

Thick Wall Cylinder: The second example case is the stress distribution in a thick wall cylinder. Due to the symmetry, the structure can be modeled by the thin wedge segment shown in Figure 9. The boundary condition, with pressure load on the inner radius, is zero displacement in the tangential direction and freedom to move in the radial direction (symmetry condition). The finite element elastic stress results for the configuration shown in Figure 9 are compared with the exact plane-strain thick wall cylinder solution in Figure 10.

If the stress conditions in the cylinder are sufficiently large, yielding will occur. A closed form solution was obtained by Mendleson (Reference 8) based on the Tresca yield criteria (i.e., $\sigma_\theta - \sigma_r > \sigma_0$). Yielding under conditions of internal pressure will occur from the inner wall to some radius $\rho = \frac{r}{R_i} = \rho_c$. The plastic and elastic stress distributions, according to Reference 8, based on bi-linear material behavior are as follows:

$$\left.\begin{aligned} \frac{\sigma_r}{\sigma_0} &= \frac{C_2}{\rho_2}\left[C_1(\rho^2-1) - \frac{p}{\sigma_0}\right] + C_3\left(\ln\rho - \frac{p}{\sigma_0}\right) \\ \frac{\sigma_\theta}{\sigma_0} &= \frac{C_2}{\rho_2}\left[C_1(\rho^2+1) + \frac{p}{\sigma_0}\right] + C_3\left(1+\ln\rho - \frac{p}{\sigma_0}\right) \end{aligned}\right\} \rho \leq \rho_c \qquad (40)$$

$$\left.\begin{aligned}
\frac{\sigma_r}{\sigma_0} &= C_4\left[\ln\rho_c - \frac{1-\beta_c^{\,2}}{2\,\beta^2_c}\;\frac{p}{\sigma_0}\right] - \frac{p}{\sigma_0\rho^2} + C_1\left(1-\frac{1}{\rho^2}\right) \\
\frac{\sigma_\theta}{\sigma_0} &= C_4\left[\ln\rho_c - \frac{1-\beta_c^{\,2}}{2\,\beta_c^{\,2}}\;\frac{p}{\sigma_0}\right] + \frac{p}{\sigma_0\rho^2} + C_1\left(1+\frac{1}{\rho^2}\right)
\end{aligned}\right\}\quad \rho > \rho_c \tag{41}$$

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) \quad \Big\} \quad \text{all } \rho \tag{42}$$

where,

$$\left.\begin{aligned}
C_1 &= \frac{\rho_c^{\,2}}{2}\;\frac{p}{\sigma_0} \quad , \quad C_2 = \frac{m(1-\nu^2)}{(1-\nu^2 m)} \\
C_3 &= \frac{1-m}{1-\nu^2 m} \quad , \quad C_4 = \frac{1-m}{(1-\nu^2)m}
\end{aligned}\right\} \tag{43}$$

The quantity β is R_o/R_i and the value of ρ_c is computed from Equation (44) below:

$$\frac{p}{\sigma_0} = \frac{\beta^2 - 1}{\beta^2}\;C_2\left[\frac{\rho_c^{\,2}}{2} + C_3\left(\ln\rho_c - \frac{1-\beta_c^{\,2}}{2\beta_c^{\,2}}\right)\right] \tag{44}$$

Stress values for the configuration shown in Figure 9 were obtained by the finite element method and by closed form solution with results shown in Figure 11.

The difference between the two sets of results is due to the different yield criteria employed. Recall that RETSCP uses the Von Mises yield criteria; whereas, the closed form solution is based on the Tresca criteria.

Specific input data for the thick wall cylinder example is given in Appendix D along with the computed results. Note, that the elastic solution is generated as a by-product of the plastic analysis (first iteration). Summary data only is given for iteration numbers 2, 3, 4, and 5.

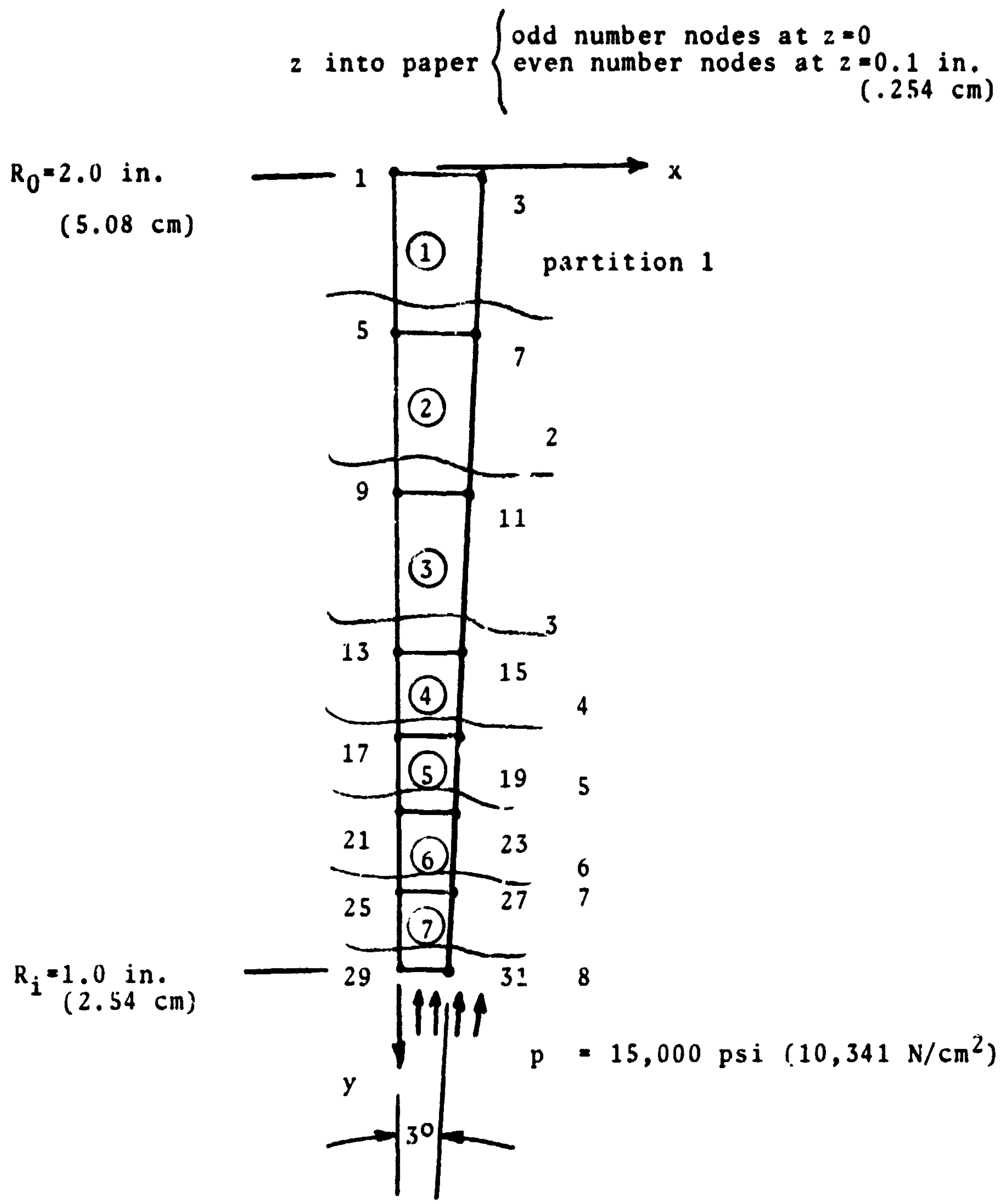


Figure 9. Configuration for thick wall cylinder example.

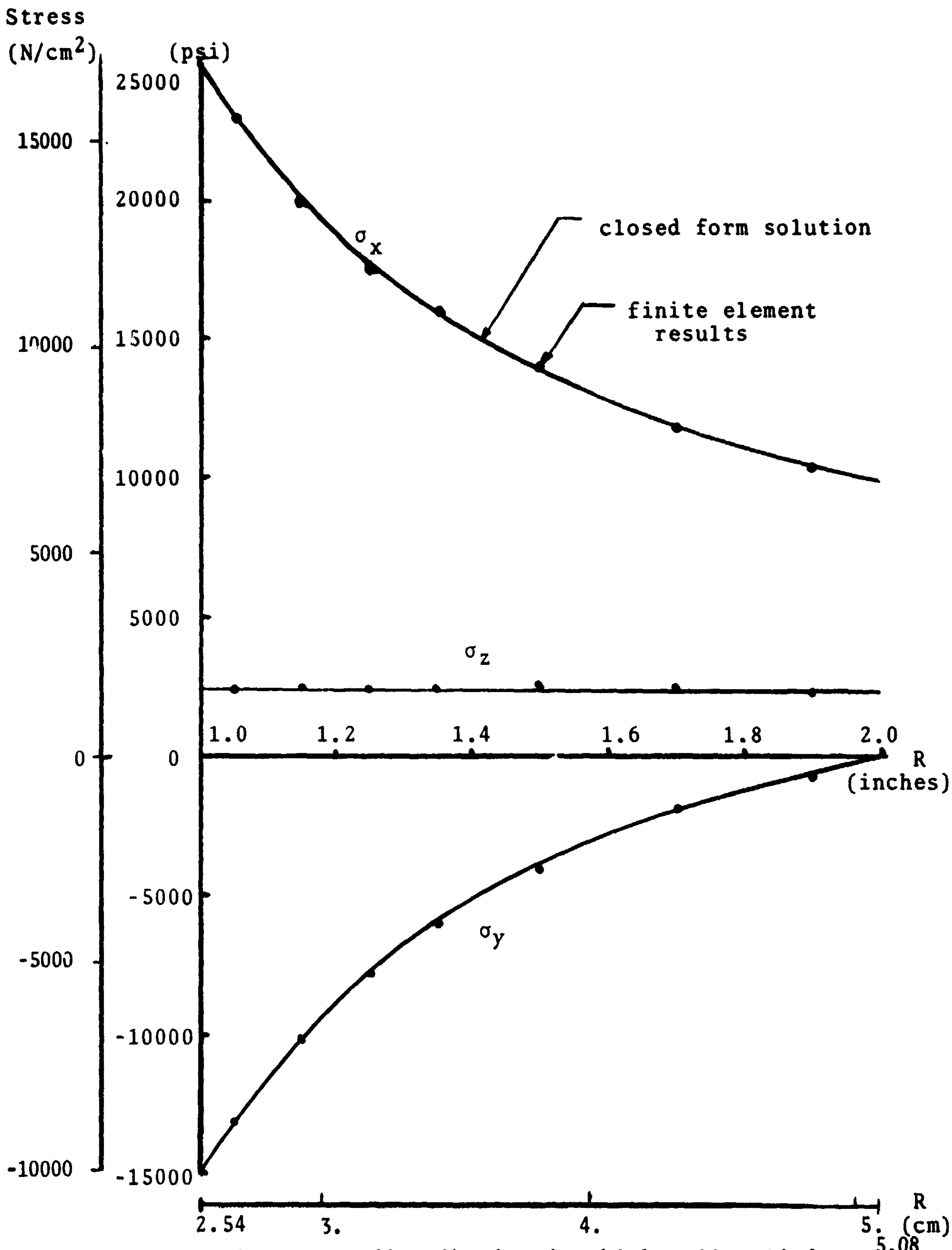


Figure 10. Elastic stress distribution in thick wall cylinder.

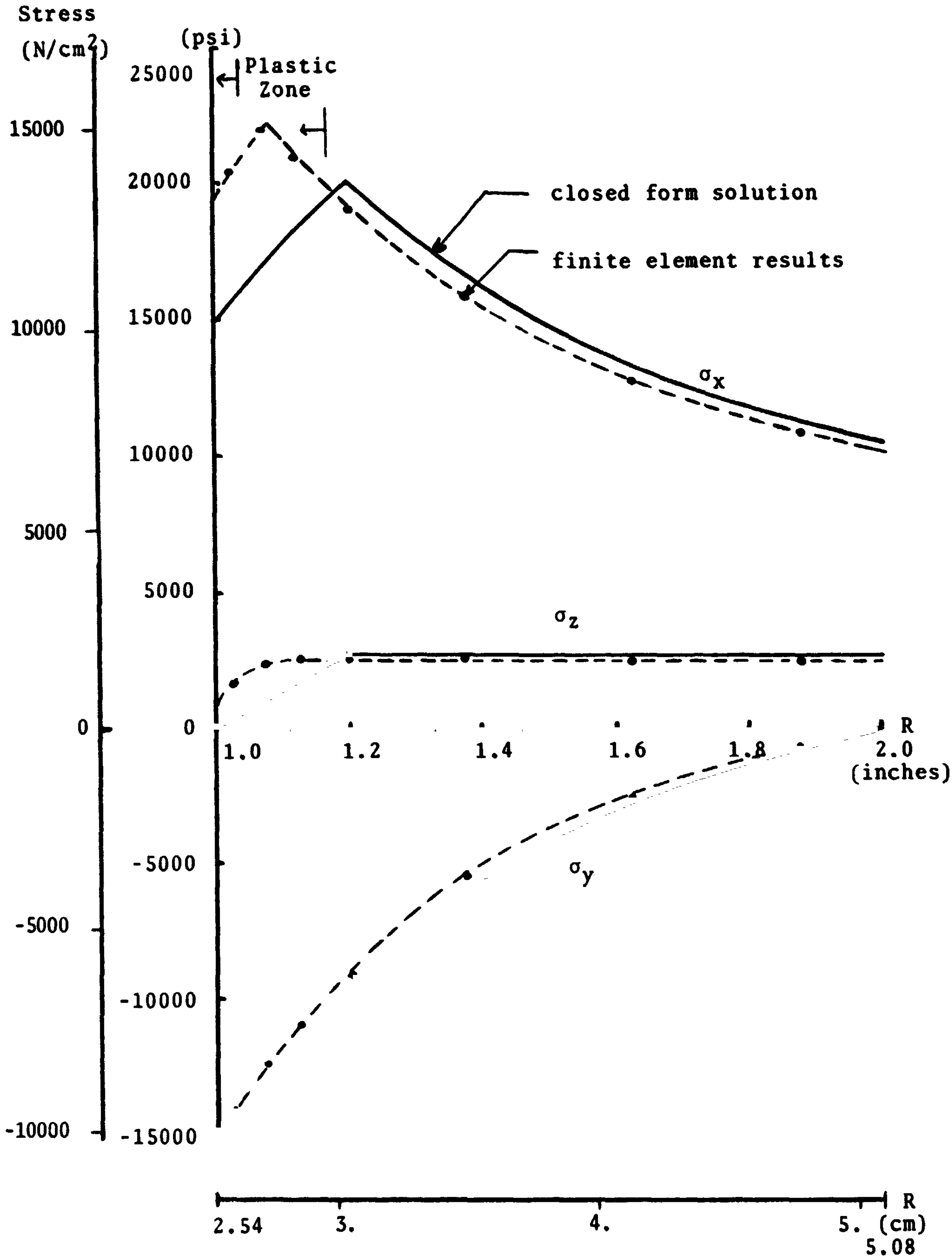


Figure 11. Stress distribution in plastic thick wall cylinder.

Heated Element Cycling: As a final example we consider a single cubic element which is cycled between two temperature limits. Two opposite faces of the cube are fixed. The temperature range is sufficiently great that the element stress exceeds the yield stress. Thus, this is an example of plastic thermal strain cycling.

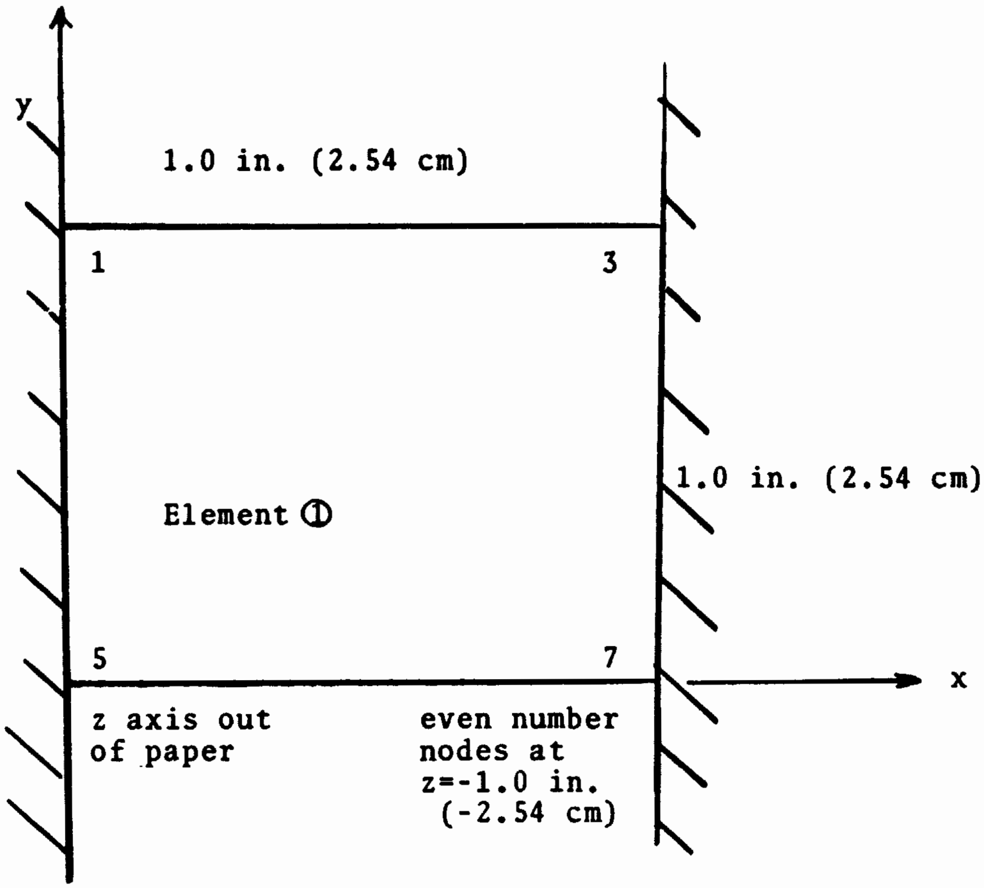
The simple finite element model is shown in Figure 1. A sample of the data input and output are given in Appendix E. The corresponding stress-strain loop is depicted graphically in Figure 13.

As the material is cooled from its stress free state, elastic stresses are built up until the yield point is reached (point "a" in Figure 13). Continued cooling causes plastic strain to the level indicated by point "b". The total strain at "b" corresponds to the cooling thermal strain. The point "c" corresponds to the plastic strain residual due to cooling.

The point "c" is the starting point for the heating cycle. As the material is heated, elastic changes occur along the line c-d. Plastic changes due to heating occur along the line d-e-f. Point "e" corresponds to the residual stress state at the original reference temperature. Thus, the plastic strain resulting from the cooling half cycle is the prescribed displacement for a subsequent analysis.

Upon heating the cube, we follow the plastic strain line d-e to point "f". The plastic strain at "g" then gives rise to the residual stress state "i" as the material returns to its original temperature.

For multi-element structures, the residual stress-strain levels during plastic cycling are determined by inputing the plastic strain values of all elements and solving the residual stress equations for the assembly.



σ_0 = 5600 psi (3,861 N/cm^2)

m = 4.04×10^{-3}

E = 17.65×10^6 psi (12.17×10^6 N/cm^2)

ν = .33

α = 9.8×10^{-6} in/in/^{o}F (17.7×10^{-6} cm/cm/^{o}C)

ΔT_{hot} = +200^{o}F (+111^{o}C)

ΔT_{cold} = -200^{o}F (-111^{o}C)

Figure 12. Configuration for heated element cycling example.

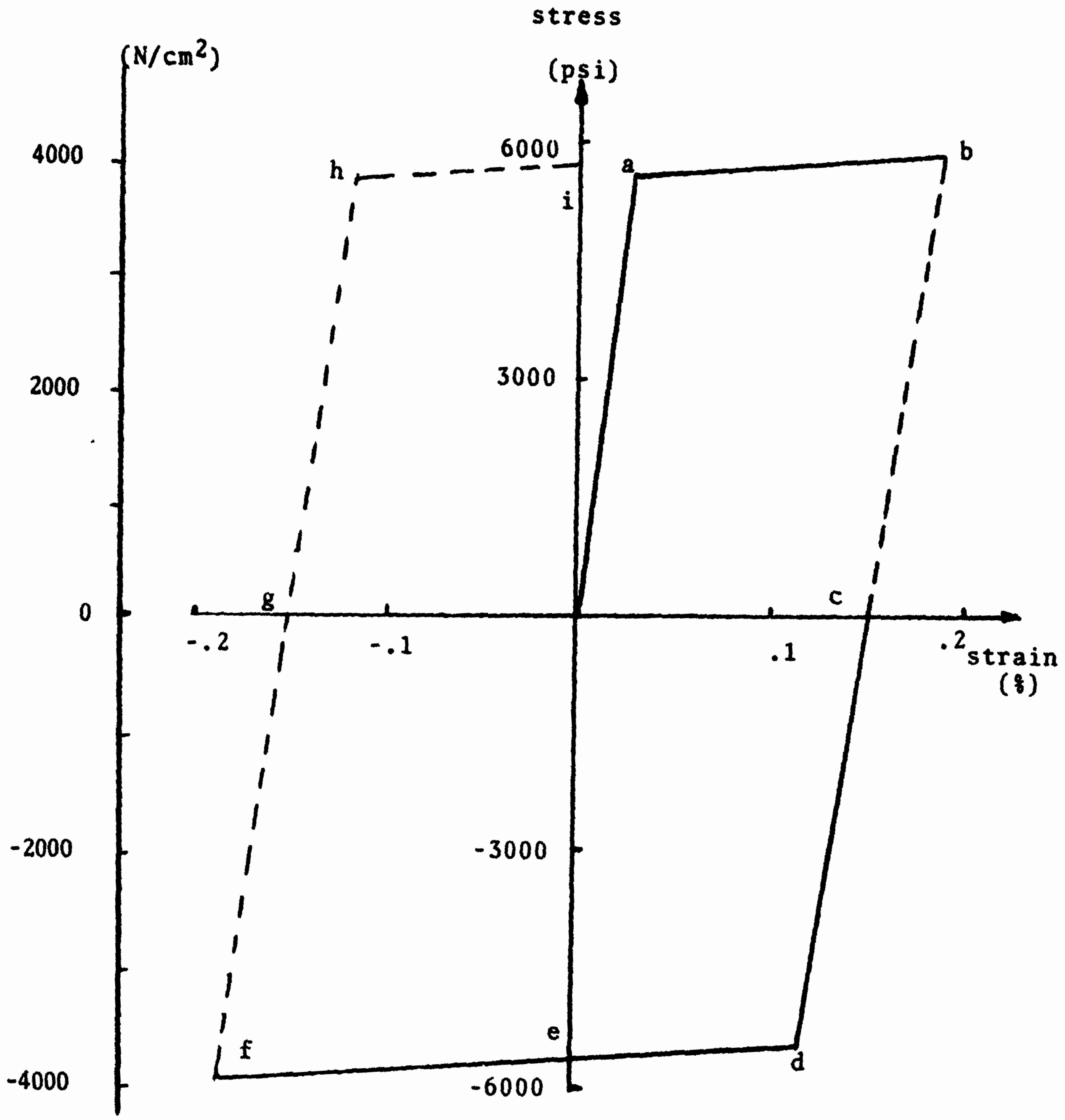


Figure 13. Stress-strain loop for heated element cycling example.

APPENDIX A--SYMBOLS

B	Matrix of differential functions, Eq. (2), (10)
D	Elastic matrix, Eq. (7)
E	Modulus of Elasticity
E_{sec}	Secant modulus, Eq. (33)
F_j	Force at nodal point j
$\bar{F}_j$	Modified force vector, Eq. (19)
F^*	Equivalent force vector, Eq. (27) (in Gaussian elimination method)
F'	Force vector in skew coordinate system
J	Jacobian matrix, Eq. (11)
K	Master stiffness matrix, Eq. (1)
K^*	Equivalent stiffness matrix, Eq. (27) (in Gaussian elimination method)
$\bar{K}$	Partitioned stiffness matrix elements
k_{ji}	Element stiffness, Eq. (18)
L	Length, or transformation matrix for skew coordinate system, Eq. (21)
m	Plastic modulus ratio
m_o	Plastic modulus ratio at reference temperature times 10^3
N_n	Parametric functions at nodal point n, Eq. (5)
P	Load
p	Pressure

APPENDIX A--SYMBOLS, Cont'd

R_i	Inner radius
R_o	Outer radius
r	Arbitrary radius
r_c	Radius at yield surface
T	Temperature
u_n	Displacement of nodal point n in x-direction
u'_n	Displacement of nodal point n in x'-direction
dV	Differential element of volume
v_n	Displacement of nodal point n in y-direction
v'_n	Displacement of nodal point n in y'-direction
w_n	Displacement of nodal point n in z-direction
x,y,z	Cartesian coordinate system
x',y'	Skew coordinate system (rotated by angle θ from x-y)

APPENDIX A--SYMBOLS, Cont'd

α	Thermal expansion coefficient
α_j	jth prescribed displacement, Eq. (19)
β	Ratio R_o/R_i, Eq. (44)
β_c	Ratio R_o/r_c
$\gamma_{xy}, \gamma_{yz}, \gamma_{xz}$	Shear strains components, Eq. (8)
$\gamma_{xy}{}^p$	Plastic shear strain, Eq. (35)
Δ	Displacement in the partitioned matrix, Eq. (26)
δ	Displacement matrix, Eq. (1), (2)
δ'	Displacement in the skew coordinate system, Eq. (23)
δ_n	Displacement at the nodal point n, Eq. (18)
ε	Strain matrix, Eq. (2)
ε_p	Plastic strain, Fig. 3
ε_{total}	Total strain, Eq. (33)
ε_{et}	Equivalent total strain, Eq. (36)
$\varepsilon_x{}^p, \varepsilon_y{}^p, \varepsilon_z{}^p$	Components of plastic strain in x, y, z directions
ξ, η, ζ	Parametric coordinate system, Fig. 1
θ	Angle of rotation of x-axis into the x'-axis in the skew coordinate system
ν	Poisson's ratio
ν_{sec}	Secant Poisson's ratio, Eq. (34)
σ	Stress

APPENDIX A--SYMBOLS, Cont'd.

σ_e	Effective stress, Eq. (32)
σ_y	Yield stress
σ_{new}	New stress, Eq. (33)
σ_o	Yield stress at reference temperature, Eq. (40)
σ_r	Radial stress component
σ_θ	Tangential stress component
ρ	Dimensionless ratio r/R_i
ρ_c	r/R_i where yield occurs at r
τ_{xy}	Shear stress component, Eq. (35)
λ_1	Yield temperature gradient
λ_2	Plastic modulus temperature gradient times 10^6 ($1/^oF$)

Special Symbols:

[ ], { }	Matrices
$[\ ]^T$	Transposed matrix form
$\|J\|$	Determinant value of J matrix
$[\ ]^{-1}$	Inverse matrix

APPENDIX B--RETSCP PROGRAM LISTING

```
$ID    Y0R6446  H PRICE                          000277     141055     12/27/73      000277
$TCP           TIME=2,PAGES=15
$MAJUND        NOPRINT
$MINUND        NOPRINT
$IBJCB         DEBUG
$IBFTC FETSCP  DECK
C
C ROCKET ENGINE THERMAL STRESS AND CYCLIC PLASTICITY
C
C
C GENERAL ELECTRIC 3-D ISOPARAMETRIC FINITE ELEMENT PROGRAM
C
C
C G.E. PROGRAM EXPANDED FOR 70 PRESCRIBED DISPLACEMENT NODES
C      AND MODIFIED TO ALLOW NON-ZERO PRESCRIBED DISPLACEMENTS.
C      ADDITIONAL OUTPUT INCLUDES THE FORCE COMPONENTS AT EACH NODE.
C  EXPANDED VERSION
C      9 PARTISIONS
C     25 NODES PER PARTISION
C     32 ELEMENTS PER PARTISION
C    225 TOTAL NODES
C     96 TOTAL ELEMENTS
C
C SECANT MODULUS PLASTICITY ITTERATION BASED ON AVE VALUE OF EFFECTIVE
C     STRESS PER ELEMENT
C
C
C
C
CMAIN        3-D ISOPARAMETRIC FINITE ELEMENTS 6/5/71  S.LEVY
C  PROGRAM NAME ISO-FINITE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C  NPART -TOTAL NUMBER OF PARTITIONS, MAXIMUM 5
C  MAXIMUM NUMBER OF NODES PER PARTITION IS 14
C  MAXIMUM NUMBER OF ISOPARAMETRIC, QUADRILATERAL ELEMENTS PER
C  PARTITION IS 16
C  NPOIN -TOTAL NUMBER OF NODAL POINTS, MAXIMUM 70
C  NELEM -TOTAL NUMBER OF ELEMENTS, MAXIMUM 32
C  NPOUN -TOTAL NUMBER OF NODAL POINTS WITH PRESCRIBED DISPLACEMENTS,
C      MAXIMUM 15
C  NVM -TOTAL NUMBER OF DIFFERENT ELASTIC PROPERTIES, MAXIMUM 2
C  NFREE -NUMBER OF DEGREES OF FREEDOM PER NOD.
C  NCARD -NUMBER OF CARDS READ IN FOR THE PREVIOUS SET,USED IN CHECKING
C  NCONC -NUMBER OF POINTS WITH CONCENTRATED LOADS
C  X  -X,Y,Z CO-ORDINATES OF THE NODAL POINTS.
C  NOD -THE 8 NODAL NUMBERS DEFINING AN ELEMENT
C  NEP -ELASTIC PROPERTY NUMBER RELEVANT TO THE ELEMENT
C  NF(I) -NODAL POINT NUMBER I WITH PRESCRIBED DISPLACEMENTS,MAXIMUM 9
C  NB -NB(I,1) = 0, DISPLACEMENT IN X DIRECTION IS PRESCRIBED
C        NB(I,2) = 0, DISPLACEMENT IN Y DIRECTION IS PRESCRIBED
C        NB(I,3) = 0, DISPLACEMENT IN Z DIRECTION IS PRESCRIBED
C        NB(I,1) = 1, DISPLACEMENT IN X DIRECTION IS NOT PRESCRIBED
C        NB(I,2) = 1, DISPLACEMENT IN Y DIRECTION IS NOT PRESCRIBED
C        NB(I,3) = 1, DISPLACEMENT IN Z DIRECTION IS NOT PRESCRIBED
C   BV -BV(I,1) = PRESCRIBED VALUE OF DISPLACEMENT IN X DIRECTION
```

```
C       RV(I,2) = PRESCRIBED VALUE OF DISPLACEMENT IN Y DIRECTION
C       RV(I,3) = PRESCRIBED VALUE OF DISPLACEMENT IN Z DIRECTION
C  NSTART -FIRST ELEMENT IN EACH PARTITION
C  NEND -LAST ELEMENT IN EACH PARTITION
C  NFIRST -FIRST NODAL POINT IN EACH PARTITION
C  NLAST -LAST NODAL POINT IN EACH PARTITION
C  U -LOADS IN X,Y AND Z DIRECTIONS
C  E1-YOUNG'S MODULUS
C  P1-POISSON'S RATIO
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      COMMON NPART,NPOIN,NELEM,NBOUN,NYM,NFREE,NCONC,
     1NPOIN2,NSTART(9),NEND(9),NFIRST(9),NLAST(9),LINES,NCY
     2 ,UTHT(675),SYLD(96),EM(96),ESEC(96),EMOD(96),EW(96),EWSEC(96)
     3,NITX,NITS,NITE,NDP,NF(225),RV(225,3),U(3,225),NB(225,3),X(225,3)
     4,NODX(96,8),A2L(96),TEMP(96),ALPHA(225),EPL(96,3)
      DIMENSION XE(8,3),NOD(8)
      NITX=1
      REWIND 1                                                    2
      REWIND 2                                                    3
      REWIND 7                                                    4
      REWIND 8                                                    5
      REWIND 9                                                    6
      CALL ISOPAR                                                 7
   40 REWIND 2                                                    8
      REWIND 3                                                    9
      REWIND 4                                                    10
      CALL READIN                                                 11
      REWIND 2                                                    12
      REWIND 4                                                    13
      CALL FACE                                                   14
      REWIND 4                                                    15
      CALL MATRIX                                                 16
      REWIND 3                                                    17
      REWIND 4
C
C     SOLUTION OF TRIDIAGONAL MATRICES
C                                                                 18
      CALL SOLVE                                                  19
      REWIND 3
C
C     CALCULATION OF STRESSES
C                                                                 20
      CALL STRESS                                                 21
      IF (NITS) 100,100,60
   60 IF (NITS-2)100,80,65
   65 DO 75 KK=1,NELEM
      READ (9,99) NN,ESEC(NN),EWSEC(NN)                           29
   75 WRITE (6,99) NN,ESEC(NN),EWSEC(NN)                          33
   80 DO 95 NITX=NITS,NITE
      REWIND 3                                                    41
      REWIND 4                                                    42
      REWIND 7                                                    43
      REWIND 8                                                    44
      REWIND 9                                                    45
      DO 90 LK=1,NELEM
      DO 85 I=1,8
```

```
      RETSCP     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      JJ=NCDX(LK,I)
      NOD(I)=JJ
      DO 85 IX=1,2
   85 XE(I,IX)=X(JJ,IX)
   90 CALL FEM3E(XE,ESEC(LK),EWSEC(LK),NOD,LK,A2L(LK),TEMP(LK),EPL)          65
      REWIND 2                                                                67
      REWIND 4                                                                68
      CALL FACE                                                               69
      REWIND 4                                                                70
      CALL MATRIX                                                             71
      REWIND 3                                                                72
      REWIND 4                                                                73
      CALL SOLVE                                                              74
      REWIND 3                                                                75
      CALL STRESS                                                             76
   95 CONTINUE
      IF (NOP) 100,100,96
   96 CONTINUE
      DO 98 NN=1,NLEM
      PUNCH 99 ,NN,ESEC(NN),EWSEC(NN)                                         84
   98 WRITE (6,99) NN,ESEC(NN),EWSEC(NN)                                      87
   99 FORMAT (I6,F20.2,F10.4)
  100 STOP
      END
```

```
$IBFTC XMATM    DECK
      SUBROUTINE MATM(D,B,DB,L,M,N)
C
C  MATRIX MULTIPLICATION  (DB)(LXN)=D(LXM)B(MXN)
C
      DIMENSION D(L,M),B(M,N),DB(L,N)
      DO 110 J=1,N
      DO 110 I=1,L
      DB(I,J)=0.
      DO 110 K=1,M
  110 DB(I,J)=DB(I,J)+D(I,K)*B(K,J)
      RETURN
      END
```

```
$IBFTC XZEROM  DECK
      SUBROUTINE  ZEROM(A,I,K)
C
C             SUBROUTINE ZEROM
C
      DIMENSION  A(1)
      II=I*K
      DO 10  J=1,II
   10 A(J)=0.0
      RETURN
      END
```

$ORIGIN ALPHA

```
$IBFTC XSOPAR  DECK
      SUBROUTINE ISOPAR
C
CISOPAR         ISOPARAMETRIC 8-NODE BOX S.LEVY 6/7/71
C
      COMMON NPART,NPCIN,NELEM,NBOUN,NYM,NFREE,NCONC,
     1NPOIN2,NSTAPT(9),NEND(9),NFIRST(9),NLAST(9),LINES,NCY
     2 ,UTHT(675),SYLD(96),EM(96),ESEC(96),EMOD(96),EW(96),EWSEC(96)
     3,NITX,NITS,NITE,NDP,NF(225),BV(225,3),U(3,225),NB(225,3),X(225,3)
     4,NODX(96,8),A2L(96),TEMP(96),ALPHA(225),EPL(96,3)
      DIMENSION A(8,3),AMX(8,3),APX(8,3),AJN(8,11,3)
   30 CONTINUE
      GAUSS=0.57735026
      DO 10 K=1,4
      DO 10 L=1,3
   10 A(K,L)=-GAUSS
      DO 11 K=1,2
      A(K+2,1)=GAUSS
   11 A(K+1,2)=GAUSS
      DO 12 K=1,4
      A(K+4,1)=A(K,1)
      A(K+4,2)=A(K,2)
   12 A(K+4,3)=-A(K,3)
      DO 13 K=1,8
      DO 13 L=1,3
      AMX(K,L)=1.0-A(K,L)
   13 APX(K,L)=1.0+A(K,L)
      DO 14 K=1,8
      AJN(K,1,1)=-0.125*AMX(K,2)*AMX(K,3)
      AJN(K,2,1)=-0.125*APX(K,2)*AMX(K,3)
      AJN(K,3,1)=+0.125*APX(K,2)*AMX(K,3)
      AJN(K,4,1)=+0.125*AMX(K,2)*AMX(K,3)
      AJN(K,5,1)=-0.125*AMX(K,2)*APX(K,3)
      AJN(K,6,1)=-0.125*APX(K,2)*APX(K,3)
      AJN(K,7,1)=+0.125*APX(K,2)*APX(K,3)
      AJN(K,8,1)=+0.125*AMX(K,2)*APX(K,3)
      AJN(K,1,2)=-0.125*AMX(K,1)*AMX(K,3)
      AJN(K,2,2)=+0.125*AMX(K,1)*AMX(K,3)
      AJN(K,3,2)=+0.125*APX(K,1)*AMX(K,3)
      AJN(K,4,2)=-0.125*APX(K,1)*AMX(K,3)
      AJN(K,5,2)=-0.125*AMX(K,1)*APX(K,3)
      AJN(K,6,2)=+0.125*AMX(K,1)*APX(K,3)
      AJN(K,7,2)=+0.125*APX(K,1)*APX(K,3)
      AJN(K,8,2)=-0.125*APX(K,1)*APX(K,3)
      AJN(K,1,3)=-0.125*AMX(K,1)*AMX(K,2)
      AJN(K,2,3)=-0.125*AMX(K,1)*APX(K,2)
      AJN(K,3,3)=-0.125*APX(K,1)*APX(K,2)
      AJN(K,4,3)=-0.125*APX(K,1)*AMX(K,2)
      AJN(K,5,3)=+0.125*AMX(K,1)*AMX(K,2)
      AJN(K,6,3)=+0.125*AMX(K,1)*APX(K,2)
      AJN(K,7,3)=+0.125*APX(K,1)*APX(K,2)
   14 AJN(K,8,3)=+0.125*APX(K,1)*AMX(K,2)
      DO 15 K=1,8
      AJN(K, 9,1)=-2.*A(K,1)
      AJN(K,10,1)=0.0
```

```
    XSOPAR     -  EFN   SOURCE STATEMENT  -  IFN(S)  -

      AJN(K,11,1)=0.0
      AJN(K, 9,2)=0.0
      AJN(K,10,2)=-2.*A(K,2)
      AJN(K,11,2)=0.0
      AJN(K, 9,3)=0.0
      AJN(K,10,3)=0.0
      AJN(K,11,3)=-2.*A(K,3)
   15 CONTINUE
      WRITE(1)(((AJN(K,L,M),K=1,8),L=1,11),M=1,3)                140
      RETURN
      END
```

$ORIGIN ALPHA

```
$IBFTC XREAD   DECK

      SUBROUTINE READIN
C
CREADIN      READING DATA AND COMPUTING STIFFNESS
CREADIN       S LEVY JUNE 8, 1971
C
      COMMON NPART,NPOIN,NELEM,NBOUN,NYM,NFREE,NCONC,
     1NPOIN2,NSTART(9),NEND(9),NFIRST(9),NLAST(9),LINES,NCY
     2 ,UTHT(675),SYLD(96),EM(96),ESEC(96),EMOD(96),EW(96),EWSEC(96)
     3,NITX,NITS,NITE,NDP,NF(225),BV(225,3),U(3,225),NB(225,3),X(225,3)
     4,NODX(96,8),A2L(96),TEMP(96),ALPHA(225),EPL(96,3)
      DIMENSION NOD(8),E1(4),P1(4),XE(8,3),A1(4),S1(4),S2(4),EM1(4),EM2(4)
     14)
   10 FORMAT (11I4)
      READ(5,10)NPART,NPOIN,NELEM,NBOUN,NYM,NFREE,NCONC,NITS,NITE,NDP
     1       ,NCY                                                    1
   11 FORMAT (9I4,F10.3)
      WRITE(6,10)NPART,NPOIN,NELEM,NBOUN,NYM,NFREE,NCONC,NITS,NITE,NDP
     1       ,NCY                                                    13
   14 DO 30  I=1,NPOIN
      READ(5,35)(X(I,J),J=1,3)                                       17
   30 WRITE(6,37)I,(X(I,J),J=1,3)                                    22
   35 FORMAT (3F16.4)
   37 FORMAT (I4,3F16.4)
   38 FORMAT (4F16.4)
   39 FORMAT (I4,4F16.4)
      READ (5,10) NCARD                                              28
      IF (NCARD-NPOIN) 110,111,110
  110 STOP
  111 CONTINUE
      DO 60 I=1,NPART
      READ (5,10) NSTART(I),NEND(I),NFIRST(I),NLAST(I)               35
   60 WRITE(6,10) NSTART(I),NEND(I),NFIRST(I),NLAST(I)               40
      DO 64  I=1,NYM
      READ(5,38)E1(I),P1(I) ,A1(I)                                   45
      READ (5,38) S1(I),S2(I),EM1(I),EM2(I)                          53
      WRITE (6,39) I,S1(I),S2(I),EM1(I),EM2(I)                       58
   64 WRITE(6,39) I,E1(I),P1(I),A1(I)                                63
      IF (NCY) 201,201,200
  200 CONTINUE
      DO 250 I=1,NELEM
      READ (5,260) IX,EPL(I,1),EPL(I,2),EPL(I,3)                     73
  250 WRITE(6,260) IX,EPL(I,1),EPL(I,2),EPL(I,3)                     78
  260 FORMAT (I6,3F15.8)
  201 CONTINUE
   24 DO 80 LK=1,NELEM
      READ (5,11) (NOD(J),J=1,8),NEP    ,TEMP(LK)                    88
      WRITE (6,11) (NOD(J),J=1,8),NEP   ,TEMP(LK)                    95
      DO 85 I=1,8
      JJ=NOD(I)
      NODX(LK,I)=NOD(I)
      DO 85 IX=1,3
   85 XE(I,IX) = X(JJ,IX)
      A2L(LK)=A1(NEP)
      CALL FEM3D (XE,E1(NEP),P1(NEP),NOD,LK,A1(NEP),TEMP(LK),EPL)    117
```

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR

```
         XREAD      H PRICE                                                        12/27/73     000277      PAGE  16
     XREAD      -   EFN   SOURCE STATEMENT  -  IFN(S)  -

      SYLD(LK)=S1(NEP)-S2(NEP)*TEMP(LK)
      EM(LK)=.001*EM1(NEP)-.000001*EM2(NEP)*TEMP(LK)
      EMOD(LK)=E1(NEP)
      ESEC(LK)=EMOD(LK)
      EW(LK)=P1(NEP)
   80 CONTINUE
      READ (5,10) NCARD                                                        138
      IF (NCARD-NELEM) 110,121,110
  121 CONTINUE
      DO 50 I=1,NBOUN
      READ (5,46) NF(I),(NB(I,J),J=1,3),(BV(I,J),J=1,3),ALPHA(I)               144
   50 WRITE(6,46) NF(I),(NB(I,J),J=1,3),(BV(I,J),J=1,3),ALPHA(I)               155
   46 FORMAT (4I4,4F16.8)
      CALL ZEROM(U,3,225)                                                      168
      IF (NCONC) 1,1,2
    2 CONTINUE
      DO 69 I=1,NCONC
      READ (5,37)K,U(1,K),U(2,K),U(3,K)                                        174
   69 WRITE(6,37)K,U(1,K),U(2,K),U(3,K)                                        179
    1 CONTINUE
      RETURN
      END
```

```
$IEFTC XFEM3C  DECK
      SUBROUTINE FEM3C(X,E1,PR1,NODE,LK,A1T,TEMX,EP)
C
CFEM3D        FEM3D ISO-PARAMETRIC, S. LEVY JUNE 6, 1971
C                         AFTER CLOUGH
C
C  X CONTAINS COORDINATES OF 8 NODES AT THE CORNERS OF THE ELEMENT.
C  NODES 5 TC 8 GO CLOCKWISE WHEN LOOKING DOWN ON THE BOX TOP.
C  NODES 1 TO 4 ARE ON THE BOTTOM BELOW 5 TO 8 RESPECTIVELY.
C  E1 MODULUS
C  PR1 POISSON'S RATIO
C  C STIFFNESS MATRIX
C
      DIMENSION CC(33,33),C(24,24),X(8,3),D(6,6),B(6,33),
     1A(6,33),NODE(8),DZA(3,3),TP(3,11),AJM(3,11,8),
     2C3(33,33),C4(9,9),C5(9,24),C6(9,24),C7(6,24)
     3  ,UTH(24),TDIS(24),SNOT(6),EPSNOT(6),EP(96,3)
      REWINC 1                                                     1
      REWINC 2
C
C  NCW TO GET THE D MATRIX.
C
      CALL ZEROM(D,6,6)                                            2
      CALL ZEROM(EPSNOT,1,6)                                       3
      CALL ZEROM(  SNOT,1,6)                                       5
      EPS=A1T*TEMX*.000001                                         7
      EPSNOT(1)=EPS  +EP(LK,1)*.01
      EPSNOT(2)=EPS  +EP(LK,2)*.01
      EPSNOT(3)=EPS  +EP(LK,3)*.01
      TA=1.0-PR1
      TB=TA-PR1
      TC=E1*TA/(TB*(1.0+PR1))
      D(1,1)=TC
      D(1,3)=TC*PR1/TA
      D(3,1)=D(1,3)
      D(2,2)=D(1,1)
      D(2,1)=D(1,3)
      D(1,2)=D(1,3)
      D(3,3)=D(1,1)
      D(2,3)=D(1,2)
      D(3,2)=D(1,2)
      D(4,4)=E1/(2.0*(1.0+PR1))
      D(5,5)=D(4,4)
      D(6,6)=D(4,4)
      WRITE (5) ((D(I,J),I=1,6),J=1,6),(EPSNOT(J),J=1,6)           12
      CALL ZEROM(C3,33,33)                                         28
  300 READ(1)(((AJM(L,K,J),J=1,8),K=1,11),L=1,3)                   29
      DO 200 NGAUSS=1,8
    5 CONTINUE
      CALL MATM(AJM(1,1,NGAUSS),X,DZA,3,8,3)                       47
    4 CONTINUE
```

```
      CALL MTINVB(D7A,3,DTRM)                                    49
    3 CONTINUE
      CALL MATM(D7A,AJM(1,1,NGAUSS),TP,3,3,11)
C
C  NOW TO GET THE B MATRIX
C                                                                52
  202 CONTINUE
      CALL ZEROM (B,6,33)                                        54
      DO 413 J=1,11
      IF(J.LE.8) K=J+2
      IF(J.GT.8) K=J-9
      B(1,3*K+1)=TP(1,J)
      B(2,3*K+2)=TP(2,J)
      B(3,3*K+3)=TP(3,J)
      B(4,3*K+1)=TP(2,J)
      B(4,3*K+2)=TP(1,J)
      B(5,3*K+2)=TP(3,J)
      B(5,3*K+3)=TP(2,J)
      B(6,3*K+1)=TP(3,J)
  413 B(6,3*K+3)=TP(1,J)
   20 CONTINUE
C  ***
C  NOW WE FORM THE STIFFNESS MATRIX C
C  ***
      CALL MATM(D,B,A,6,6,33)                                    83
      WRITE(2) ((B(I,J),I=1,6),J=1,33)                           84
  126 CALL MATTM(B,A,CC,33,6,33)                                 93
      DO 40 J=1,33
      DO 40 K=1,33
   40 C3(J,K)=C3(J,K)+CC(J,K)*DTRM
 1001 CONTINUE
  200 CONTINUE
      DO 100 K=1,8
  100 BACKSPACE 2                                                112
      DO 414 K=1,9
      DO 414 J=1,9
  414 C4(J,K)=C3(J,K)
 1002 CONTINUE
      CALL MTINVB(C4,9,DTRM)                                     127
 1003 CONTINUE
      DO 415 J=1,9
      DO 415 K=1,24
  415 C5(J,K)=C3(J,K+9)
      CALL MATM(C4,C5,C6,9,9,24)                                 140
 1004 CONTINUE
      CALL MATTM(C5,C6,C,24,9,24)                                142
 1005 CONTINUE
      DO 420 J=1,24
      DO 420 K=1,24
  420 C(J,K)=-C(J,K)+C3(J+9,K+9)
      NI=NODE(1)
      TDIS(1)=0.
      TDIS(2)=0.
      TDIS(3)=0.
      K=3
      DO 2100 I=2,8
```

```
      NT=NODE(I)
      DO 2100 J=1,3
      K=K+1
 2100 TDIS(K)=    (X(I,J)-X(1,J))*EPSNOT(J)
      CALL MATM(C,TDIS,UTH,24,24,1)                                 173
      WRITE (8) (UTH(J),J=1,24),(NODE(J),J=1,8)                     174
 1006 CONTINUE
      WRITE(3) ((C(J,K),J=1,24),K=1,24),(NODE(J),J=1,8),LK          187
  419 DO 600 NGAUSS=1,8
  418 CONTINUE
      READ(2) ((A(I,J),I=1,6),J=1,33)                               207
      CALL MATM(A,C6,C7,6,9,24)                                     215
      DO 416 J=1,6
      DO 416 K=1,24
  416 C7(J,K)=-C7(J,K)+A(J,K+9)
      WRITE(4) ((C7(I,J),I=1,6),J=1,24),(X(NGAUSS,I),I=1,
     U3),(NODE(I),I=1,8),LK                                         226
  600 CONTINUE
 1000 RETURN
      END
```

```
$IBFTC YMATTM DECK
C
      SUBROUTINE MATTM (D,B,DB,L,M,N)
C  MATRIX MULTIPLICATION TRANSPOSED (DB)(L X N)=D(M X L)B(M X N)
C
      DIMENSION D(M,L),B(M,N),DB(L,N)
      DO 110 J=1,N
      DO 110 I=1,L
      DB(I,J)=0.
      DO 110 K=1,M
  110 DB(I,J)=DB(I,J)+D(K,I)*B(K,J)
      RETURN
      END
```

```
$IBFTC XMTINV  DECK
      SUBROUTINE MTINVD(A,N,DETERM)
C
C           MATRIX INVERSION WITH VALUE OF DETERMINANT 6/9/71
C
C  A IS MATRIX BEING INVERTED
C  N IS MATRIX SIZE
      DIMENSION IPIVOT(9),A(N,N),INDEX(9,2),PIVOT(9)
C
C     INITIALIZATION
C
   10 DETERM=1.0
   15 DO 20 J=1,N
   20 IPIVOT(J)=0
   30 DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
   40 AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVOT(J)-1)60,105,60
   60 DO 100 K=1,N
   70 IF (IPIVOT(K)-1)80,100,740
   80 IF (ABS(AMAX)-ABS(A(J,K)))85,100,100
   85 IROW=J
   90 ICOLUM=K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLUM)140,260,140
  140 DETERM=-DETERM
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
  310 PIVOT(I)=A(ICOLUM,ICOLUM)
  320 IF(N.LT.4)  DETERM=DETERM*PIVOT(I)
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
C
C     REDUCE NON-PIVOT ROWS
```

```
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM)400,550,400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I
  620 IF (INDEX(L,1)-INDEX(L,2))630,710,630
  630 JROW=INDEX(L,1)
  640 JCOLUM=INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLUM)
  700 A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
  740 RETURN
  750 END
```

$ORIGIN ALPHA

```
$IBFTC XFACE   DECK
      SUBROUTINE FACE
C
C                  FACE STRESS COMBINATION, JUNE 6,1971, S. LEVY
C
      COMMON NPART,NPOIN,NELEM,NBOUN,NYM,NFREE,NCONC,
     1NPOIN2,NSTART(9),NEND(9),NFIRST(9),NLAST(9),LINES,NCW
     2 ,UTHT(675),SYLD(96),EM(96),ESEC(96),EMOD(96),EW(96),EWSEC(96)
     3,NITX,NITS,NITE,NDP,NF(225),BV(225,3),U(3,225),NB(225,3),X(225,3)
     4,NODX(96,8),A2L(96),TEMP(96),ALPHA(225),EPL(96,3)
      DIMENSION DDBA(6,24,8),DBA(6,24),XX(3,8),Z(3),NCODE(8),NFACE(4,6)
     1,DBB(6,24),Y(3),DSUM(6,24)
      NFACE(1,1)=1
      NFACE(2,1)=2
      NFACE(3,1)=3
      NFACE(4,1)=4
      NFACE(1,2)=5
      NFACE(2,2)=6
      NFACE(3,2)=7
      NFACE(4,2)=8
      NFACE(1,3)=1
      NFACE(2,3)=2
      NFACE(3,3)=5
      NFACE(4,3)=6
      NFACE(1,4)=8
      NFACE(2,4)=3
      NFACE(3,4)=4
      NFACE(4,4)=7
      NFACE(1,5)=5
      NFACE(2,5)=1
      NFACE(3,5)=8
      NFACE(4,5)=4
      NFACE(1,6)=3
      NFACE(2,6)=2
      NFACE(3,6)=7
      NFACE(4,6)=6
   21 DO 200 NGAUSS=1,8
  200 READ(4) ((DDBA(I,J,NGAUSS),I=1,6),J=1,24),(XX(I,NGAUSS),I=1,3),      6
     1  (NODE(I),I=1,8),LL                                                25
      CALL ZEROM(DSUM,6,24)
      DO 300 NX=1,6,2
      N1=NFACE(1,NX)
      N2=NFACE(2,NX)
      N3=NFACE(3,NX)
      N4=NFACE(4,NX)
      N5=NFACE(1,NX+1)
      N6=NFACE(2,NX+1)
      N7=NFACE(3,NX+1)
      N8=NFACE(4,NX+1)
      DO 301 J=1,6
      DO 301 K=1,24
      DBB(J,K)=0.25*(DDBA(J,K,N5)+DDBA(J,K,N6)+DDBA(J,K,N7)+DDBA(J,K,N8)
     1)
  301 DBA(J,K)=0.25*(DDBA(J,K,N1)+DDBA(J,K,N2)+DDBA(J,K,N3)+
     1    DDBA(J,K,N4))
```

```
       DO 302 J=1,3
       Y(J)=0.25*(XX(J,N5)+XX(J,N6)+XX(J,N7)+XX(J,N8))
 302   Z(J)=C.25*(XX(J,N1)+XX(J,N2)+XX(J,N3)+XX(J,N4))
       DO 303 J=1,6
       DO 303 K=1,24
       TA=1.366*DBA(J,K)-.366*DBB(J,K)
       DBB(J,K)=-.366*DBA(J,K)+1.366*DBB(J,K)
 303   DBA(J,K)=TA
       WRITE(2) ((DBA(I,J),I=1,6),J=1,24),(Z(I),I=1,3),NODE(N1),NODE(N2),
      1 NODE(N3),NODE(N4),LL,(NODE(I),I=1,8)                              89
       WRITE(2) ((DBB(I,J),I=1,6),J=1,24),(Y(I),I=1,3),NODE(N5),NODE(N6),
      1NODE(N7),NODE(N8),LL,(NODE(I),I=1,8)                               109
 300   CONTINUE
       DO 320 I=1,6
       DO 320 J=1,24
       DO 310 NG=1,8
 310   DSUM(I,J)=DSUM(I,J)+.125*DDBA(I,J,NG)
 320   CONTINUE
       WRITE (2) ((DSUM(I,J),I=1,6),J=1,24)
       IF(LL .NE.NELEM) GO TO 21                                          147
 100   CONTINUE
       RETURN
       END
```

$ORIGIN ALPHA

```
$IBFTC XMATRIX DECK

      SUBROUTINE MATRIX
C
CMATRIX        FORMATION OF MATRICES - S.LEVY, 6/4/71
C
      COMMON NPART,NPOIN,NELEM,NBOUN,NYM,NFREE,NCONC,
     1NPOIN2,NSTART(9),NEND(9),NFIRST(9),NLAST(9),LINES,NCY
     2 ,UTHT(675),SYLD(96),EM(96),ESEC(96),EMOD(96),EW(96),EWSEC(96)
     3,NITX,NITS,NITE,NDP,NF(225),BV(225,3),U(3,225),NB(225,3),X(225,3)
     4,NODX(96,8),A2L(96),TEMP(96),ALPHA(225),EPL(96,3)
      DIMENSION UU(75),NODE(8),C(24,24),UUU(75),ST(75,150),UTH(24)
      REWIND 8                                                   1
      CALL ZEROM(UTHT,1,675)                                     2
      DO 10 NX=1,NELEM
      READ (8) (UTH(J),J=1,24),(NODE(J),J=1,8)                   6
      L=0
      DO 10 J=1,8
C PUT THERMAL LOAD INTO ROTATED SYSTEM
      DO 13 NZ=1,NBOUN
      IF (NODE(J)-NF(NZ)) 13,12,13
   12 NJZ=3*(J-1)
      ALP=ALPHA(NZ)/57.2958
      UONE=UTH(NJZ+1)
      UTWO=UTH(NJZ+2)
      UTH(NJZ+1)=UONE*COS(ALP)+UTWO*SIN(ALP)                     28    29
      UTH(NJZ+2)=-UONE*SIN(ALP)+UTWO*COS(ALP)                    31    32
   13 CONTINUE
C COMPLETE
      DO 10 K=1,3
      L=L+1
      J5=3*(NODE(J)-1)+K
   10 UTHT(J5)=UTHT(J5)+UTH(L)
   37 FORMAT(I4,3F16.4)
      INTER = 0
      CALL ZEROM(UUU,1,75)                                       49
      DO 70 II=1,NPART
      REWIND 3                                                   53
      CALL ZEROM(ST,75,150)                                      54
  975 CONTINUE
      NST=NSTART(II)
      NEN=NEND(II)
      K=NFIRST(II)
      L=NLAST(II)
      IF(II.NE.NPART)  KEND=NLAST(II+1)
      IF(II.EQ.NPART)KEND=NLAST(II)
      MINUS = K-1
      LMINUS=3*(L-MINUS)
      DO 80 LK=1,NELEM
      MM=LK-INTER
   82 READ(3) ((C(J,I),J=1,24),I=1,24),(NODE(I),I=1,8),NL       74
      IF(NL.LT.NST) GO TO 80
      IF(NL.GT.NEN) GO TO 80
  884 CONTINUE
      DO 800 LL=1,8
      DO 800 KK=1,8
```

```
      XMATRI      -  EFN    SOURCE  STATEMENT  -  IFN(S)  -

      IF (NODE(KK)-K) 900,131,131
  131 IF (NODE(KK)-L) 132,132,900
  132 M=NFREE*(NODE(KK)-K)
      N=NFREE*(NODE(LL)-K)
      I=NFREE*(KK-1)
      J=NFREE*(LL-1)
      IF (N) 800,900,900
  900 DO 5 NJ=1,NFREE
      DO 5 MI=1,NFREE
      MMI=M+MI
      NNJ=N+NJ
      IMI=I+MI
      JNJ=J+NJ
    5 ST(MMI,NNJ) = ST(MMI,NNJ) + C(IMI,JNJ)
  800 CONTINUE
   80 CONTINUE
 980  CONTINUE
      MI1=NFREE*MINUS+1
      NJ1=NFREE*L
      M1=NJ1-MI1+1
      IF (II-NPART) 9115,9116,9115
 9115 NA1=NFREE*(NLAST(II+1)-MINUS)
      GO TO 9117
 9116 NA1=M1+1
 9117 N1=NA1-M1
      MM1=M1+1
C ST IS PUT INTO ROTATED SYSTEM
      DO 440 NZ=K,L
      DO 440 NZC=1,NBOUN
      IF (NZ-NF(NZC)) 440,405,440
  405 NJZ=3*(NZ-K)
      ALP=ALPHA(NZC)/57.2958
      DO 470 NZZ=1,NA1
      STONE=ST(NJZ+1,NZZ)
      STTWO=ST(NJZ+2,NZZ)
      ST(NJZ+1,NZZ)=STONE*COS(ALP)+STTWO*SIN(ALP)                    155   156
      ST(NJZ+2,NZZ)=-STONE*SIN(ALP)+STTWO*COS(ALP)                   158   159
  470 CONTINUE
  440 CONTINUE
      DO 480 NZ=K,KEND
      DO 480 NZC=1,NBOUN
      IF (NZ-NF(NZC)) 480,450,480
  450 NJZ=3*(NZ-K)
      ALP=ALPHA(NZC)/57.2958
      DO 490 NZZ=1,M1
      STONE=ST(NZZ,NJZ+1)
      STTWO=ST(NZZ,NJZ+2)
      ST(NZZ,NJZ+1)=STONE*COS(ALP)+STTWO*SIN(ALP)                    179   180
      ST(NZZ,NJZ+2)=-STONE*SIN(ALP)+STTWO*COS(ALP)                   182   183
  490 CONTINUE
  480 CONTINUE
C EVERYTHING BELOW IS IN ROTATED SYSTEM
      WRITE (7) M1,N1,MM1,NA1,((ST(I,J),I=1,M1),J=1,M1),
     1    ((ST(I,J),I=1,M1),J=MM1,NA1)                              189
      JNJ=0
      DO 981 J=K,L
```

```
      XMATRI     -  EFN    SOURCE STATEMENT  -  IFN(S)  -
      DO 981 I=1,3
      JNJ=JNJ+1
      JS=3*(J-1)+I
  981 UU(JNJ)=UUU(JNJ)+U(I,J)+UTHT(JS)
      CALL ZEROM(UUU,1,75)
C
C     INTRODUCTION OF PRESCRIBED DISPLACEMENTS                    217
C
      DO 290 I=1,NBOUN
      M=NF(I)-K
      MM=NF(I)-1
      KKEND=KEND-NF(I)
      IF (M) 290,242,242
  242 IF(KKEND) 290,243,243
  243 DO 230 J=1,NFREE
      IF (NB(I,J)) 230,345,230
  345 NMI = NFREE*M+J
      LLEAR=NFREE*(L-K+1)
      DO 1345 KLEAR=1,LLEAR
      JNJ=KLEAR
      UU(JNJ)=UU(JNJ)-ST(KLEAR,NMI)*BV(I,J)
 1345 CONTINUE
      IF (II-NPART) 1233,239,239
 1233 IF (NPART-1) 1231,239,1231
 1231 LEA=LLEAR+1
 1232 CONTINUE
      IF (NMI-LLEAR) 1234,1234,239
 1234 NMX=NMI
      KLEP=0
      DO 1235 KLE=LEA,NA1
      KLEP=KLEP+1
 1235 UUU(KLEP)=UUU(KLEP)-ST(NMX,KLE)*BV(I,J)
  239 CONTINUE
 7345 CONTINUE
  230 CONTINUE
  290 CONTINUE
      DO 4347 I=1,NBOUN
      M=NF(I)-K
      KKEND=KEND-NF(I)
      IF (M) 4347,4242,4242
 4242 IF (KKEND) 4347,4243,4243
 4243 DO 4247 J=1,NFREE
      IF (NB(I,J)) 4247,4344,4247
 4344 NMI=NFREE*M+J
      LLEAR=NFREE*(L-K+1)
      DO 4345 KLEAR=1,LLEAR
      JNJ=KLEAR
      IF (KLEAR.EQ.NMI) UU(JNJ)=BV(I,J)
      ST(KLEAR,NMI)=0.
      IF (KLEAR.NE.NMI) GO TO 4345
      LLR=(KEND-K+1)*NFREE
      DO 4346 KKL=1,LLR
      ST(NMI,KKL)=0.
 4346 CONTINUE
      ST(NMI,NMI)=1.
 4345 CONTINUE
```

```
4247 CONTINUE
4347 CONTINUE
     INTER=NEN
     MI=NFREE*MINUS+1
     NJ=NFREE*L
     M=NJ-MI+1
     IF (II-NPART)115,116,115
 115 NA=NFREE*(NLAST'II+1)-MINUS)
     GO TO 117
 116 NA=M+1
 117 N=NA-M
     MM=M+1
   8 FORMAT (I5,8E13.4)
   7 FORMAT (I5,E13.4)
  70 WRITE(4)M,N,((ST(I,J),I=1,M),J=1,M),((ST(I,J),I=1,M),J=MM,NA),              322
    1(UU(I),I=1,M)
   3 FORMAT (1H1 10X 3HII= I4,6X 2HM= I4, 6X 2HN= I4   /// )
   4 FORMAT (10X 5HCHECK /// )
     RETURN
     END
```

$ORIGIN ALPHA

```
$IBFTC XSOLVE  DECK
      SUBROUTINE SOLVE
C
CSOLVE      SOLUTION OF EQUATIONS S LEVY 6/10/71
C
      COMMON NPART,NPOIN,NELEM,NBOUN,NYM,NFREE,NCONC,
     1NPOIN2,NSTART(9),NEND(9),NFIRST(9),NLAST(9),LINES,NCY
     2 ,UTHT(675),SYLD(96),EM(96),ESEC(96),EMOD(96),EW(96),EWSEC(96)
     3,NITX,NITS,NITE,NDP,NF(225),BV(225,3),U(3,225),NB(225,3),X(225,3)
     4,NODX(96,8),A2L(96),TEMP(96),ALPHA(225),EPL(96,3)
      DIMENSION AM(75,75),BM(75,75),YM(75,75),TF(75),DIS(75),F(75),
     1      XF(75),YF(75),ZF(75),FIS(75)
      NSIZE=75
      CALL ZEROM(AM,NSIZE,NSIZE)                                    3
      CALL ZEROP(TF,1,NSIZE)                                        5
      DO 144 LL=1,NPART
      READ(4) M,N,((YM(I,J),I=1,M),J=1,M),((BM(I,J),I=1,M),J=1,N),
     1(F(I),I=1,M)                                                  9
  150 DO 426 I=1,M
      F(I)=F(I)-TF(I)
      DO 424 J=1,M
  424 AM(I,J)=YM(I,J)-AM(I,J)
  426 CONTINUE
      CALL MTINVC(AM,M,NSIZE)
C
C     MATRIX INVERSION PROGRAM
C
      WRITE(2) M,N,((AM(I,J),I=1,M),J=1,M),((BM(I,J),I=1,M),J=1,N), 45
     1(F(I),I=1,M)
      CALL MATMS(AM,F,DIS,M,M,NSIZE)                                46
      IF (NPART-LL) 437,437,432                                     65
  432 CALL MATTMS(BM,DIS,TF,N,M,NSIZE)                              65
      DO 110 J=1,N
      DO 110 I=1,M
      YM(I,J)=0.0
      DO 110 K=1,M
  110 YM(I,J)=YM(I,J)+AM(I,K)*BM(K,J)
      DO 111 J=1,N
      DO 111 I=1,N
      AM(I,J)=0.0
      DO 111 K=1,M
  111 AM(I,J)=AM(I,J)+BM(K,I)*YM(K,J)
  144 CONTINUE
  437 WRITE (3) (DIS(I),I=1,M)                                     104
      IF (NPART-1) 600,600,601
  601 NA=NPART-1
      DO 441 LL=1,NA
      BACKSPACE 2                                                  116
      BACKSPACE 2                                                  117
      READ (2) M,N,((AM(I,J),I=1,M),J=1,M),((BM(I,J),I=1,M),J=1,N),
     1(F(I),I=1,M)                                                 118
      CALL MATMS(BM,DIS,TF,M,N,NSIZE)                              139
      DO 444 I=1,M
  444 F(I)=F(I)-TF(I)
      CALL MATMS(AM,F,DIS,M,M,NSIZE)                               148
```

```
      XSOLVE    -  EFN   SOURCE STATEMENT  -  IFN(S)  -
  441 WRITE (3) (DIS(I),I=1,M)
  600 CONTINUE                                                          149
C
C
C COMPUTE NODAL FORCES
C
C
      WRITE (6,1116)                                                    156
      WRITE (6,1112)                                                    157
      REWIND 7                                                          158
      BACKSPACE 3                                                       159
      READ (7) M1,N1,MM1,NA1,((AM(I,J),I=1,M1),J=1,M1),
     1      ((BM(I,J),I=1,M1),J=1,N1)                                   160
      READ (3) (DIS(I),I=1,M1)                                          183
      IX=0
      CALL MATMS(AM,DIS,XF,M1,M1,NSIZE)                                 151
      IF (NPART-1) 699,667,670
  667 KF=M1-2
      DO 668 K=1,KF,3
      IX=IX+1
      IY=3*(IX-1)
      XF(K)=XF(K)-UTHT(IY+1)
      XF(K+1)=XF(K+1)-UTHT(IY+2)
      XF(K+2)=XF(K+2)-UTHT(IY+3)
  668 WRITE (6,1111)  IX,DIS(K),DIS(K+1),DIS(K+2),XF(K),XF(K+1),XF(K+2)   210
      GO TO 699
  670 CALL MATTMS(BM,DIS,TF,N1,M1,NSIZE)                                221
      CALL ZEROM(ZF,1,NSIZE)                                            223
      DO 675 K=1,M1
  675 FIS(K)=DIS(K)
      DO 695 II=2,NPART
      BACKSPACE 3                                                       235
      BACKSPACE 3                                                       236
      READ (3) (DIS(I),I=1,N1)                                          237
      CALL MATMS(BM,DIS,YF,M1,N1,NSIZE)                                 242
      DO 681 K=1,M1
  681 F(K)=XF(K)+YF(K)+ZF(K)
      KF=M1-2
      DO 683 K=1,KF,3
      IX=IX+1
      IY=3*(IX-1)
       F(K)= F(K)-UTHT(IY+1)
      F(K+1)=F(K+1)-UTHT(IY+2)
      F(K+2)=F(K+2)-UTHT(IY+3)
  683 WRITE (6,1111)  IX,FIS(K),FIS(K+1),FIS(K+2),F(K),F(K+1),F(K+2)      266
  685 CONTINUE
      DO 686 K=1,N1
      FIS(K)=DIS(K)
  686 ZF(K)=TF(K)
      READ (7) M1,N1,MM1,NA1,((AM(I,J),I=1,M1),J=1,M1),
     1      ((BM(I,J),I=1,M1),J=1,N1)                                   282
      CALL MATMS(AM,DIS,XF,M1,M1,NSIZE)                                 301
      IF (NPART-II) 699,695,690
  690 CALL MATTMS(BM,DIS,TF,N1,M1,NSIZE)                                305
  695 CONTINUE
      DO 696 K=1,M1
```

```
 696 F(K)=XF(K)+ZF(K)
     KF=M1-2
     DO 697 K=1,KF,3
     IX=IX+1
     IY=3*(IX-1)
      F(K)= F(K)-UTHT(IY+1)
     F(K+1)=F(K+1)-UTHT(IY+2)
     F(K+2)=F(K+2)-UTHT(IY+3)
 697 WRITE (6,1111)  IX,DIS(K),DIS(K+1),DIS(K+2),F(K),F(K+1),F(K+2)                    333
1111 FORMAT (I5,6E13.4)
1116 FORMAT (1H1 10X 14HROTATED SYSTEM    )
1112 FORMAT (     2X //// 6H  NODE 6X 7HX-DISPL 6X 7HY DISPL 6X 7HZ-DISPL
    1L 6X 7HX-FORCE 6X 7HY-FORCE 6X 7HZ-FORCE  // )
 699 CONTINUE
     RETURN
     END
```

```
$IBFTC XMTIN   DECK
      SUBROUTINE MTINVC(A,N,NSIZE)
C
C  MATRIX INVERSION, MODIFIED 6/8/71 BY S. LEVY
C  A IS MATRIX BEING INVERTED
C  N IS MATRIX SIZE
C  NSIZE IS MEMORY SIZE
C
      DIMENSION A(NSIZE,NSIZE)
  3C0 DO 55C ICOLUM=1,N
  310 PIVOTI=1.C/A(ICOLUM,ICOLUM)
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM)=1.0
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)*PIVOTI
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 55C L1=1,N
  39C IF(L1-ICOLUM)4CC,550,4C0
  400 T=A(L1,ICOLUM)
  42C A(L1,ICOLUM)=C.0
  43C DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  550 CONTINUE
    0 RETURN
  550 END
```

```
$IBFTC XMATMS  DECK
      SUBROUTINE MATMS(D,B,DB,L,M,NSIZE)
C
C  MATRIX MULTIPLICATION   DB(L)=D(LXM)*B(M)
C
      DIMENSION D(NSIZE,NSIZE),B(NSIZE),DB(NSIZE)
C  NSIZE IS MEMORY SIZE
      DO 110 I=1,L
      DB(I)=0.
      DO 110 K=1,M
  110 DB(I)=DB(I)+D(I,K)*B(K)
      RETURN
      END
```

```
$IBFTC XMATTM  DECK

      SUBROUTINE MATTMS(D,B,DB,L,M,NSIZE)
C
C  MATRIX MULTIPLICATION TRANSPOSED DB(L)=D(MXL)*B(M)
C
      DIMENSION D(NSIZE,NSIZE),B(NSIZE),DB(NSIZE)
C  NSIZE IS MEMORY SIZE
      DO 110 I=1,L
      DB(I)=0.
      DO 110 K=1,M
  110 DB(I)=DB(I)+D(K,I)*B(K)
      RETURN
      END
```

$ORICIN ALPHA

```
$IBFTC XSTRES  DECK

      SUBROUTINE STRESS
C
C STRESS        CALCULATION OF  STRESSES.
C
      COMMON NPART,NPOIN,NELEM,NBOUN,NYM,NFREE,NCONC,
     1NPOIN2,NSTART(9),NEND(9),NFIRST(9),NLAST(9),LINES,NCY
     2 ,UTHT(675),SYLD(96),EM(96),ESEC(96),EMOD(96),EW(96),EWSEC(96)
     3,NITX,NITS,NITE,NDP,NF(225),RV(225,3),U(3,225),NB(225,3),X(225,3)
     4,NODX(56,8),A2L(96),TEMP(96),ALPHA(225),EPL(96,3)
      DIMENSION V(675),DBA(6,24),NODE(8),DEF(24),SIG(6),NODD(4),SIGE(96)
     1,SNOT(6),DSUM(6,24),D(6,6),SX(96),SY(96),EPSNOT(6)
      DO 600 II=1,NPART
      JJ=NPART+1-II
      M=NFREE*(NFIRST(JJ)-1) + 1
      N=NFREE*NLAST(JJ)
  600 READ (3) (V(I),I=M,N)
C ROTATE DISPLACEMENTS BACK TO X - Y - Z                              9
      DO 550 NZ=1,NBOUN
      NJZ=3*(NF(NZ)-1)
      ALP=ALPHA(NZ)/57.2958
      VONE=V(NJZ+1)
      VTWO=V(NJZ+2)
      V(NJZ+1)=VONE*COS(ALP)-VTWO*SIN(ALP)                          24   25
      V(NJZ+2)=VONE*SIN(ALP)+VTWO*COS(ALP)                          27   28
  550 CONTINUE
C COMPLETE
  614 FORMAT (1H1 10X )
      WRITE (6,614)                                                 32
      WRITE (6,615)                                                 33
  615 FORMAT(//5H NODE,16H X-DISPLACEMENTS,16H Y-DISPLACEMENTS,16H Z-DIS
     1PLACEMENTS//)
      WRITE (6,32) ( I,V(3*I-2),V(3*I-1),V(3*I) ,I=1,NPOIN)         34
   32 FORMAT(1H ,I4,3E16.8)
      WRITE (6,614)                                                 43
      WRITE (6,625)                                                 44
  625 FORMAT(//16H ELEMENT NUMBER ,8X,17HFACE NODE NUMBERS,8X,
     122H X,Y AND Z COORDINATES)
      WRITE (6,635)                                                 45
  635 FORMAT(4X,16H  X-STRESS      ,16H  Y-STRESS      ,
     1    16H   Z-STRESS      ,16H  XY-STRESS      ,16H  YZ-STRESS      ,
     2    16H XZ-STRESS       )
      REWIND 2                                                      46
      REWIND 5                                                      47
      L=0
   21 CONTINUE
      L=L+1
      SIGE(L)=0.
      READ (5) ((D(I,J),I=1,6),J=1,6) (EPSNOT(J),J=1,6)             51
      DO 200 JZ=1,6
      READ(2) ((DBA(I,J),I=1,6),J=1,24),ORX,ORY,ORZ,(NODD(I),I=1,4),
     1LL,(NODE(I),I=1,8)                                            65
  622 DO 620 I=1,8
      JS=3*(NODE(I)-1)
      JJ=NODE(I)
```

```
      DO 620 IJ=1,3
      JX=JS+IJ
      I3=I+I+I-3+IJ
      J3=JJ+JJ+JJ-3+IJ
  620 DEF(I3)=V(J3)-V(JX)
      CALL MATM(DHA,DEF,SIG,6,24,1)                                   102
  632 WRITE (6,10) LL,(NOD)(I),I=1,4),DRX,DRY,DRZ                     103
      DO 633 K=1,3
      SNOT(K+3) = SIG(K+3)
  633 SNOT(K)=SIG(K)-EPSNOT(K)
      WRITE (6,31) (SNOT(I),I=1,6)                                    116
      CALL MATM(D,SNOT,SIG,6,6,1)                                     121
      WRITE (6,31) (SIG(I),I=1,6)                                     122
  200 CONTINUE
      READ (2) ((DSUM(I,J),I=1,6),J=1,24)                             129
      CALL MATM(DSUM,DEF,SIG,6,24,1)                                  139
      DO 634 K=1,3
      SNOT(K+3) = SIG(K+3)
  634 SNOT(K)=SIG(K)-EPSNOT(K)
      CALL MATM(D,SNOT,SIG,6,6,1)                                     152
      WRITE (6,39) LL,(SIG(K),K=1,6)                                  153
      SX(L)=2.*SIG(1)-SIG(2)-SIG(3)
      SY(L)=2.*SIG(2)-SIG(1)-SIG(3)
      SIE=(.5*((SIG(1)-SIG(2))**2+(SIG(1)-SIG(3))**2+(SIG(2)-SIG(3))**2
     1     )+3.*(SIG(4)**2+SIG(5)**2+SIG(6)**2))**.5                  162
      SIGE(L)=SIE
   39 FORMAT ( 4X / 27H AVERAGE STRESS FOR ELEMENT  I3,/ 1X 6F16.6,/ 4X)
      IF(LL-NELEM) 21,100,100
   38 FORMAT (1H,6E16.8)
   10 FORMAT(1H ,4X,I4,11X,4I5,6X,3F14.6)
   31 FORMAT(1H ,6F16.6)
  100 CONTINUE
      WRITE (6,33) NITX                                               165
      DO 300 J=1,NELEM
      ETOT=SIGE(J) /ESEC(J)
      ESTAR=SYLD(J)/EMOD(J)
      IF (ETOT.LT.ESTAR) GO TO 350
      SIGNEW=SYLD(J)*(1.-EW(J))+EW(J)*EMOD(J)*ETOT
      ESEC(J)=SIGNEW/ETOT
      EPLAS=ETOT-SIGNEW/EMOD(J)
      EWSEC(J)=.5-(.5-EW(J))*ESEC(J)/EMOD(J)
      GO TO 330
  350 SIGNEW=SIGE(J)
      EPLAS=0.
      EWSEC(J)=EW(J)
      ESEC(J)=EMOD(J)
  330 CONTINUE
      SX(J)=50.*EPLAS*SX(J)/SIGE(J)
      SY(J)=50.*EPLAS*SY(J)/SIGE(J)
      SIGE(J)=100.*(EPLAS+2.*(1.+EW(J))*SIGE(J)/(3.*EMOD(J)))
  300 WRITE (6,34) J,ETOT,EPLAS,SIGNEW ,SYLD(J),      ESEC(J),EWSEC(J)  204
  321 FORMAT (1H1 4X /// 9H  ELEMENT 4X 16HEQUIVALENT TOTAL 4X
     1     8HPLASTIC   28HSTRAIN COMPONENTS (PERCENT) / 8H  NUMBER 4X
     2          16HSTRAIN (PERCENT) 4X 5HX-DIR 10X 5HY-DIR 10X 5HZ-DIR //)
  322 FORMAT (I6,8X F10.5,3F15.5)
  323 FORMAT (I6,3F15.8)
```

```
      IF (INITX-MITE) 320,310,320
  310 WRITE (6,321)                                                   211
      DO 315 J=1,NELEM
      SZ=-(SX(J)+SY(J))
      SX(J)=SX(J)+EPL(J,1)
      SY(J)=SY(J)+EPL(J,2)
      SZ=SZ+EPL(J,3)
      WRITE (6,322) J,SIGE(J),SX(J),SY(J),SZ                          224
      PUNCH     323, J,SX(J),SY(J),SZ                                 228
  315 CONTINUE
  320 CONTINUE
   33 FORMAT (1H1 10X /// 18H YIELD CHECK AFTER I4, 12H ITTERATIONS
     3     /// 4X 7HELEMENT 5X
     1   5HTOTAL 5X 7HPLASTIC 3X 9HEFFECTIVE 4X 5HYIELD 6X
     2   6HSECANT 7X 6HSECANT / 16X 6HSTRAIN 4X 6HSTRAIN 4X
     4   6HSTRESS 7X              6HSTRESS 5X 7HMODULUS 6X 7HPOISSON //)
   34 FORMAT (I8,F14.6,F10.6,       F11.1,F11.1,F14.1,F13.4  / )
      RETURN
      END
```

APPENDIX C--CANTILEVER BEAM EXAMPLE-
INPUT AND OUTPUT DATA

TITLE: Cantilever Beam Example | PROJECT NUMBER: 4880 | ANALYST: Ibrahim | SHEET 1 OF 2

FORTRAN STATEMENT

```
$DATA
    5 20   4    4  1   3   2    1  0   0  0
         -0.5000         0.0000          0.5000
          0.5000         0.0000          0.5000
          0.5000         0.0000         -0.5000
         -0.5000         0.0000         -0.5000
         -0.5000         1.0000          0.5000
          0.5000         1.0000          0.5000
          0.5000         1.0000         -0.5000
         -0.5000         1.0000         -0.5000
         -0.5000         2.0000          0.5000
          0.5000         2.0000          0.5000
          0.5000         2.0000         -0.5000
         -0.5000         2.0000         -0.5000
         -0.5000         3.0000          0.5000
          0.5000         3.0000          0.5000
          0.5000         3.0000         -0.5000
         -0.5000         3.0000         -0.5000
         -0.5000         4.0000          0.5000
          0.5000         4.0000          0.5000
          0.5000         4.0000         -0.5000
         -0.5000         4.0000         -0.5000
   20
    1  1   1    4
    1  2   5    8
```

NASA-C-836 (REV. 9-14-59)

TITLE: Cantilever Beam Example | PROJECT NUMBER: 4880 | ANALYST: Ibrahim | SHEET 2 OF 2

FORTRAN STATEMENT

```
    2 3    9  12
    3 4   13  16
    4 4   17  20
   30000000.0000          0.2500          0.0
      10000.              0.              0.0                 0.0
    5 6    7   8  1    2   3   4   1     0.0
    9 10  11  12  5    6   7   8   1     0.0
   13 14  15  16  9   10  11  12   1     0.0
   17 18  19  20  13  14  15  16   1     0.0
    4
    1 0    0   0      0.             0.               0.             0.
    2 0    0   0      0.             0.               0.             0.
    3 0    0   0      0.             0.               0.             0.
    4 0    0   0      0.             0.               0.             0.
   17         0.              0.              0.5000
   18         0.              0.              0.5000
```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```
UNUSED CORE                    60455 THRU  64673

  BEGIN EXECUTION.
  5  20   4   4   1   3   2   1   0   C   0
  1          -C.5000          0.          0.5000
  2           C.5000          0.          0.5000
  3           C.5000          C.         -0.5000
  4          -C.5000          0.         -0.5000
  5          -C.5000          1.0000      0.5000
  6           C.5000          1.0000      0.5000
  7           0.5000          1.0000     -0.5000
  8          -0.5000          1.0000     -0.5000
  9          -C.5000          2.0000      0.5000
 10           C.5000          2.0000      J.5000
 11           C.5000          2.0000     -0.5000
 12          -C.5000          2.0000     -0.5000
 13          -C.5000          3.0000      0.5000
 14           C.5000          3.0000      0.5000
 15           C.5000          3.0000     -0.5000
 16          -C.5000          3.0000     -0.5000
 17          -C.5000          4.0000      0.5000
 18           C.5000          4.0000      0.5000
 19           C.5000          4.0000     -0.5000
 20          -C.5000          4.0000     -0.5000
  1   1   1   4
  1   2   5   8
  2   3   9  12
  3   4  13  16
  4   4  17  20
  1        100CC.0000         0.          0.          0.
  1     300000C0.0000         C.2500      0.
  5   6   7   8   1   2   3   4   1   0.
  9  10  11  12   5   6   7   8   1   0.
 13  14  15  16   9  10  11  12   1   0.
 17  18  19  20  13  14  15  16   1   0.
  1   0   0   0     0.          0.          0.          0.
  2   0   0   0     0.          0.          0.          0.
  3   0   C   0     0.          0.          0.          0.
  4   0   0   0     0.          0.          0.          0.
 17           0.              0.          0.5000
 18           C.              0.          0.5000
```

ROTATED SYSTEM

NODE	X-DISPL	Y-DISPL	Z-DISPL	X-FORCE	Y-FORCE	Z-FORCE
1	-0.	0.	0.	0.3666E+00	0.2000E+01	-0.2526E+00
2	0.	0.	0.	-0.3656E+00	0.2000E+01	-0.2526E+00
3	0.	0.	0.	0.3666E+00	-0.2000E+01	-0.2474E+00
4	0.	0.	0.	-0.3666E+00	-0.2000E+01	-0.2474E+00
5	-0.8522E-07	-0.5774E-06	0.7603E-06	0.1192E-06	0.1073E-05	-0.8941E-06
6	0.8522E-07	-0.6774E-06	0.7603E-06	-0.8941E-07	0.6557E-06	-0.5364E-06
7	-0.8496E-07	0.6776E-06	0.7614E-06	-0.7451E-07	-0.4768E-06	-0.5960E-07
8	0.8496E-07	0.6776E-06	0.7614E-06	0.5950E-07	-0.2384E-06	0.3576E-06
9	-0.4857E-07	-0.1180E-05	0.2704E-05	0.6333E-06	0.2414E-05	-0.4232E-05
10	0.4857E-07	-0.1180E-05	0.2704E-05	-0.5737E-06	0.1520E-05	-0.2265E-05
11	-0.4959E-07	0.1180E-05	0.2699E-05	-0.4843E-06	-0.2086E-06	0.7153E-06
12	0.4960E-07	0.1180E-05	0.2699E-05	0.2235E-06	0.6855E-06	0.1609E-05
13	-0.2600E-07	-0.1478E-05	0.5436E-05	0.9015E-06	0.3934E-05	-0.6795E-05
14	0.2601E-07	-0.1478E-05	0.5436E-05	-0.1132E-05	0.3219E-05	-0.5364E-05
15	-0.2213E-07	0.1481E-05	0.5452E-05	-0.5439E-06	-0.7153E-06	0.2146E-05
16	0.2215E-07	0.1481E-05	0.5452E-05	0.1132E-05	0.3576E-06	0.4649E-05
17	-0.2961E-08	-0.1586E-05	0.8617E-05	0.3576E-06	0.2384E-06	0.5000E+00
18	0.2977E-08	-0.1586E-05	0.8617E-05	-0.5960E-07	0.	0.5000E+00
19	-0.1739E-07	0.1578E-05	0.8559E-05	-0.6258E-06	-0.	0.1192E-05
20	0.1741E-07	0.1578E-05	0.8559E-05	0.5364E-06	-0.2384E-06	0.9537E-06

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	-0.	-0.	C.
2	0.	0.	0.
3	0.	0.	C.
4	0.	0.	0.
5	-0.85219192E-07	-0.67738649E-06	0.76033275E-06
6	0.85220178E-07	-0.67738807E-06	0.76033367E-06
7	-0.8496C874E-07	0.67764499E-C6	0.76136453E-06
8	0.84963300E-07	0.67764653E-06	0.76136399E-06
9	-0.4856[illegible]12E-07	-0.11800619E-05	0.27035648E-05
10	0.4857[illegible]E-07	-0.11800646E-C5	0.27035665E-05
11	-0.4955[illegible]1E-07	0.11795480E-C5	0.26994427E-05
12	0.49602491E-07	0.11795508E-C5	0.26994415E-05
13	-0.26000833E-07	-0.14781813E-C5	C.54362533E-05
14	0.26011054E-07	-0.14781846E-05	0.54362555E-05
15	-0.22132639E-07	0.14805031E-05	C.54517196E-05
16	0.22147333E-07	0.14805064E-05	0.54517175E-05
17	-0.29605580E-08	-0.15857792E-C5	C.86171866E-05
18	0.29772913E-08	-0.15857825E-05	C.86171892E-05
19	-0.17391217E-07	0.15775343E-C5	0.85594572E-05
20	0.17412610E-07	0.15775374E-C5	0.85594548E-05

```
ELEMENT NUMBER          FACE NODE NUMBERS           X,Y AND Z COORDINATES
     X-STRESS            Y-STRESS          Z-STRESS          XY-STRESS          YZ-STRESS          XZ-STRESS
        1                 5    6    7    8          0.                 1.000000        0.
      0.000000          0.000000        -0.000000          0.000000          0.000000          0.000000
     -0.000002          0.000011        -0.030928          0.000001          0.999987          0.000000
        1                 1    2    3    4          0.                 0.              0.
      0.000000          0.000000         0.000000          0.000000          0.000000         -0.000000
      0.000004          0.000012         0.000004          0.000001          0.999987         -0.000000
        1                 5    6    1    2         -0.                 0.500000        0.500000
      0.000000         -0.000001         0.000000         -0.000000          0.000000          0.000000
     -2.697143        -20.999171        -0.015462         -0.000004          0.999987          0.000000
        1                 4    7    8    3         -0.                 0.500000       -0.500000
     -0.000000          0.000001        -0.000000          0.000000          0.000000         -0.000000
      2.697145         20.999194        -0.015462          0.000005          0.999987         -0.000000
        1                 1    5    4    8         -0.500000           0.500000        0.
      0.000000          0.000000        -0.000000          0.000000          0.000000          0.000000
      0.000001          0.000036        -0.015459          0.000001          0.999983          0.000000
        1                 7    6    3    2          0.500000           0.500000       -0.
      0.000000          0.000000        -0.000000          0.000000          0.000000         -0.000000
      0.000001         -0.000013        -0.015466          0.000001          0.999991         -0.000000

AVERAGE STRESS FOR ELEMENT  1
      0.000001          0.000011        -0.015463          0.000001          0.999987          0.

        2                 9   10   11   12          0.                 2.000000        0.
     -0.000000         -0.000000         0.000000          0.000000          0.000000          0.000000
      0.000000          0.000010         0.123707          0.000001          0.999988          0.000000
        2                 5    6    7    8          0.                 1.000000        0.
      0.000000          0.000000        -0.000000          0.000000          0.000000         -0.000000
     -0.000001          0.000010        -0.030925          0.000000          0.999989         -0.000000
        2                 9   10    5    6         -0.                 1.500000        0.500000
      0.000000         -0.000000         0.000000         -0.000000          0.000000          0.000000
      0.275244        -14.999428         0.[illegible]6391    -0.000004          0.999988          0.000000
        2                 8   11   12    7         -0.                 1.500000       -0.500000
     -0.000000          0.000000        -0.000000          0.000000          0.000000          0.
     -0.275246         14.999446         0.046390          0.000004          0.999989          0.
        2                 5    9    8   12         -0.500000           1.500000        0.
     -0.000000         -0.000000         0.000000          0.000000          0.000000          0.000000
     -0.000001          0.000028         0.046389          0.000001          0.999984          0.000000
        2                11   10    7    6          0.500000           1.500000       -0.
     -0.000000         -0.000000         0.000000          0.000000          0.000000         -0.000000
     -0.000001         -0.000007         0.046393          0.000000          0.999993         -0.000000

AVERAGE STRESS FOR ELEMENT  2
     -0.000001          0.000009         0.046391          0.0000[illegible]          0.999989          0.000000

        3                13   14   15   16          0.                 3.[illegible]0        0.
      0.000000          0.000000        -0.000000          0.000000          0.000000         -0.000000
      0.000000          0.000006        -0.463913          0.000001          0.999992         -0.000001
        3                 9   10   11   12          0.                 2.000000        0.
     -0.000000         -0.000000         0.000000          0.000000          0.000000          0.
     -0.000001          0.000005         0.123700          0.000001          0.999992          0.
        3                13   14    9   10         -0.                 2.500000        0.500000
      0.000000         -0.000000         0.000000         -0.000000          0.000000         -0.000000
     -0.055301         -8.999680        -0.170106         -0.000003          0.999992         -0.000001
        3                12   15   16   11         -0.                 2.500000       -0.500000
     -0.000000          0.000000        -0.000000          0.000000          0.000000         -0.000000
```

```
    0.055302       8.999694      -0.170104       0.000006       0.999992      -0.000000
    3              9   13   12   16          -0.500000      2.500000      0.
    0.000000       0.000000      -0.000000       0.000000       0.000000      -0.000000
    0.000000       0.000014      -0.170107       0.000001       0.999988      -0.000000
    3             15   14   11   10           0.500000      2.500000     -0.
    0.000000       0.000000      -0.000000       0.000003       0.000000      -0.000000
   -0.000000      -0.000001      -0.170104       0.000001       0.999995      -0.000001

AVERAGE STRESS FOR ELEMENT  3
   -0.000000       0.000006      -0.170106       0.000001       0.999992      -0.000000

    4             17   18   19   20           0.             4.000000      0.
   -0.000000      -0.000000       0.000000       0.000000       0.000000       0.000000
    0.000000       0.000001       1.731928       0.000000       1.000000       0.000002
    4             13   14   15   16           0.             3.000000      0.
    0.000000       0.000000      -0.000000       0.000000       0.000000      -0.000000
   -0.000001       0.000002      -0.463890       0.000000       1.000001      -0.000002
    4             17   18   13   14          -0.             3.500000      0.500000
    0.000000      -0.000000       0.000000      -0.000000       0.000000       0.000000
    0.277745      -2.999907       0.634020      -0.000001       1.000000       0.000000
    4             16   19   20   15          -0.             3.500000     -0.500000
   -0.000000       0.000000      -0.000000       0.000003       0.000000       0.000000
   -0.277744       2.959910       0.634021       0.000001       1.000001       0.000000
    4             13   17   16   20          -0.500000      3.500000      0.
   -0.000000      -0.000000       0.000000       0.000003       0.000000       0.000000
   -0.000001      -0.000001       0.634017       0.000000       0.999998       0.000001
    4             19   18   15   14           0.500000      3.500000     -0.
   -0.000000      -0.000000       0.000000       0.000003       0.000000      -0.000000
    0.000000       0.000002       0.634020       0.000000       1.000002      -0.000000

AVERAGE STRESS FOR ELEMENT  4
   -0.000001       0.000000       0.634018       0.000000       1.000000       0.000001
```

YIELD CHECK AFTER 1 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.000000	0.	1.7	10000.0	30000000.0	0.2500
2	0.000000	0.	1.7	10000.0	30000000.0	0.2500
3	0.000000	0.	1.7	10000.0	30000000.0	0.2500
4	0.000000	0.	1.8	10000.0	30000000.0	0.2500

01 EXIT IN RETSCP

APPENDIX D--THICK WALL CYLINDER EXAMPLE-
INPUT AND OUTPUT DATA

TITLE: **Thick Wall Cylinder Example** | PROJECT NUMBER: 4880 | ANALYST: Ibrahim | SHEET 1 OF 4

FORTRAN STATEMENT

```
$DATA
    8 32     7  32   1    3   4   2   7   1   0
         0.               0.              0.
         0.               0.               .1000
         0.1047           0.0027          0.
         0.1047           0.0027           .1000
         0.               0.2500          0
         0.               0.2500          0.1000
         0.0916           0.2524          0.
         0.0916           0.2524          0.1000
         0.               0.5000          0.
         0.               0.5000          0.1000
         0.0785           0.5021          0.
         0.0785           0.5021          0.1000
         0.               0.7500          0.
         0.               0.7500          0.1000
         0.0654           0.7517          0.
         0.0654           0.7517          0.1000
         0.               0.8500          0.
         0.               0.8500          0.1000
         0.0601           0.8516          0.
         0.0601           0.8516          0.1000
         0.               0.9000          0.
         0.               0.9000          0.1000
```

NASA-C-836 (REV 9-14-59)

TITLE: Thick Wall Cylinder Example

PROJECT NUMBER: 4880

ANALYST: Ibrahim

SHEET 2 OF 4

FORTRAN STATEMENT

```
    0.0576          0.9015          0.
    0.0576          0.9015          0.1000
    0.              0.9500          0.
    0.              0.9500          0.1000
    0.0550          0.9514          0.
    0.0550          0.9514          0.1000
    0.              1.0000          0.
    0.              1.0000          0.1000
    0.0523          1.0014          0.
    0.0523          1.0014          0.1000
   32
    1  1   1   4
    1  2   5   8
    2  3   9  12
    3  4  13  16
    4  5  17  20
    5  6  21  24
    6  7  25  28
    7  7  29  32
  30000000.0000     0.2500          6.5000
      30000.0000    0.              0.1000      -0
    2  4   8   6   1   3   7   5   1   0.
    6  8  12  10   5   7  11   9   1   0.
   10 12  16  14   9  11  15  13   1   0.
```

NASA-C-836 (REV 9-14-59)

TITLE: Thick Wall Cylinder Example | PROJECT NUMBER: 4880 | ANALYST: Ibrahim | SHEET 3 OF 4

FORTRAN STATEMENT

```
   14 16  20  18  13  15  19  17   1        0.
   18 20  24  22  17  19  23  21   1        0.
   22 24  28  26  21  23  27  25   1        0.
   26 28  32  30  25  27  31  29   1        0.
    7
    1  0   1   0      -0.             -0.             -0.             -0.
    2  0   1   0      -0.             -0.             -0.             -0.
    3  0   1   0      -0.             -0.             -0.             3.00000000
    4  0   1   0      -0.             -0.             -0.             3.00000000
    5  C   1   0      -0.             -0.             -0.             -0.
    6  0   1   0      -0.             -0.             -0.             -0.
    7  0   1   0      -0.             -0.             -0.             3.00000000
    8  0   1   0      -0.             -0.             -0.             3.00000000
    9  0   1   0      -0.             -0.             -0.             -0.
   10  0   1   0      -0.             -0.             -0.             -0.
   11  0   1   0      -0.             -0.             -0.             3.00000000
   12  0   1   0      -0.             -0.             -0.             3.00000000
   13  0   1   0      -0.             -0.             -0.             -0.
   14  0   1   0      -0.             -0.             -0.             -0.
   15  0   1   0      -0.             -0.             -0.             3.00000000
   16  0   1   0      -0.             -0.             -0.             3.00000000
   17  0   1   0      -0.             -0.             -0.             -0.
   18  0   1   0      -0.             -0.             -0.             -0.
   19  0   1   0      -0.             -0.             -0.             3.00000000
```

NASA-C-836 (REV 9-14-59)

TITLE	PROJECT NUMBER	ANALYST	SHEET
Thick Wall Cylinder Example	4880	Ibrahim	4 OF 4

FORTRAN STATEMENT

STATEMENT NUMBER (1–5)	CONT (6)	7–12	13–18	19–24	25–30	31–36	37–42	43–48	49–54	55–60	61–66	67–72	IDENTIFICATION (73–80)
20		0 1	0	-0.			-0.			-0.		3.	00000000
21		0 1	0	-0.			-0.			-0.		-0.	
22		0 1	0	-0.			-0.			-0.		-0.	
23		0 1	0	-0.			-0.			-0.		3.	00000000
24		0 1	0	-0.			-0.			-0.		3.	00000000
25		0 1	0	-0.			-0.			-0.		-0.	
26		0 1	0	-0.			-0.			-0.		-0.	
27		0 1	0	-0.			-0.			-0.		3.	00000000
28		0 1	0	-0.			-0.			-0.		3.	00000000
29		0 1	0	-0.			-0.			-0.		-0.	
30		0 1	0	-0.			-0.			-0.		-0.	
31		0 1	0	-0.			-0.			-0.		3.	00000000
32		0 1	0	-0.			-0.			-0.		3.	00000000
29			0.			-19.6500		-0.					
30			0.			-19.6500		-0.					
31			1.0290			-19.6200		-0.					
32			1.0290			-19.6200		-0.					

NASA-C-836 (REV 9-14-59)

```
UNUSED CORE                    60455 THRU  64673

  BEGIN EXECUTION.
  8  32    7  32    1    3    4    2    7    1    0
  1          C.                  0.              0.
  2          C.                  0.              0.1000
  3          0.1047              0.0027          0.
  4          C.1047              0.0027          0.1000
  5          C.                  C.2500          0.
  6          C.                  C.2500          0.1000
  7          C.0916              0.2524          0.
  8          0.C916              0.2524          0.1000
  9          0.                  0.5000          0.
 10          C.                  C.5000          0.1000
 11          C.0785              0.5021          0.
 12          C.C785              0.5021          0.1000
 13          C.                  0.7500          0.
 14          C.                  0.75CC          0.1000
 15          0.0654              0.7517          0.
 16          C.0654              0.7517          0.1000
 17          C.                  C.8500          0.
 18          C.                  0.8500          0.1000
 19          C.0601              0.8516          0.
 20          C.C601              0.8516          0.1000
 21          0.                  C.9000          0.
 22          0.                  C.9000          0.1000
 23          0.0576              C.9015          0.
 24          C.0576              0.9015          0.1000
 25          0.                  C.9500          0.
 26          C.                  C.9500          0.1000
 27          0.0550              C.9514          0.
 28          C.0550              C.9514          0.1000
 29          0.                  1.0000          0.
 30          C.                  1.0000          0.1000
 31          C.C523              1.0014          0.
 32          0.0523              1.0014          0.1000
  1    1    1    4
  1    2    5    8
  2    3    9   12
  3    4   13   16
  4    5   17   20
  5    6   21   24
  6    7   25   28
  7    7   29   32
  1       300CC.0000             0.              0.1000         -0.
  1    3000C0CC.0000             0.2500          6.5000
  2    4    8    6    1    3    7    5    1    0.
  6    8   12   1C    5    7   11    9    1    0.
 10   12   16   14    9   11   15   13    1    0.
 14   16   20   18   13   15   19   17    1    0.
 18   20   24   22   17   19   23   21    1    0.
 22   24   28   26   21   23   27   25    1    0.
 26   28   32   3C   25   27   31   29    1    0.
  1    0    1    0      -0.            -0.            -0.            -0.
  2    0    1    0      -0.            -0.            -0.            -0.
  3    0    1    0      -0.            -0.            -0.             3.00000000
  4    0    1    0      -0.            -0.            -0.             3.00000000
  5    0    1    0      -0.            -0.            -0.            -0.
  6    0    1    C      -C.            -0.            -0.            -0.
  7    0    1    0      -0.            -0.            -0.             3.00000000
  8    0    1    C      -0.            -0.            -0.             3.00000000
  9    0    1    0      -0.            -0.            -0.            -0.
```

10	0	1	0	-0.		-0.		-0.	-0.
11	0	1	0	-0.		-0.		-0.	3.00000000
12	0	1	0	-0.		-0.		-0.	3.00000000
13	0	1	0	-0.		-0.		-0.	-0.
14	0	1	0	-0.		-0.		-0.	-0.
15	0	1	0	-0.		-0.		-0.	3.00000000
16	0	1	0	-0.		-0.		-0.	3.00000000
17	0	1	0	-0.		-0.		-0.	-0.
18	0	1	0	-0.		-0.		-0.	-0.
19	0	1	0	-0.		-0.		-0.	3.00000000
20	0	1	0	-0.		-0.		-0.	3.00000000
21	0	1	0	-0.		-0.		-0.	-0.
22	0	1	0	-0.		-0.		-0.	-0.
23	0	1	0	-0.		-0.		-0.	3.00000000
24	0	1	0	-0.		-0.		-0.	3.00000000
25	0	1	0	-0.		-0.		-0.	-0.
26	0	1	0	-0.		-0.		-0.	-0.
27	0	1	0	-0.		-0.		-0.	3.00000000
28	0	1	0	-0.		-0.		-0.	3.00000000
29	0	1	0	-0.		-0.		-0.	-0.
30	0	1	0	-0.		-0.		-0.	-0.
31	0	1	0	-0.		-0.		-0.	3.00000000
32	0	1	0	-0.		-0.		-0.	3.00000000
29			0.		-19.6500		-0.		
30			0.		-19.6500		-0.		
31			1.0290		-19.6200		-0.		
32			1.0290		-19.6200		-0.		

NODE	X-DISPL	Y-DISPL	Z-DISPL	X-FORCE	Y-FORCE	Z-FORCE
1	0.	-0.6235E-03	-0.	-0.6489E+02	0.1907E-04	-0.1519E+02
2	-0.	-0.6235E-03	-0.	-0.6499E+02	0.1526E-04	0.1519E+02
3	-0.	-0.6235E-03	-0.	0.6487E+02	0.1144E-04	-0.1519E+02
4	-0.	-0.6235E-03	-0.	0.6487E+02	-0.5722E-05	0.1519E+02
5	0.	-0.6568E-03	-0.	-0.1449E+03	0.2289E-04	-0.2856E+02
6	-0.	-0.6568E-03	-0.	-0.1449E+03	0.3815E-04	0.2856E+02
7	-0.	-0.6568E-03	-0.	0.1449E+03	-0.1907E-05	-0.2856E+02
8	-0.	-0.6568E-03	-0.	0.1449E+03	-0.1335E-04	0.2856E+02
9	0.	-0.7099E-03	-0.	-0.1752E+03	0.2861E-04	-0.2446E+02
10	-0.	-0.7099E-03	-0.	-0.1752E+03	0.2861E-04	0.2446E+02
11	-0.	-0.7099E-03	-0.	0.1752E+03	-0.3815E-05	-0.2446E+02
12	-0.	-0.7099E-03	-0.	0.1752E+03	-0.1144E-04	0.2446E+02
13	0.	-0.7947E-03	-0.	-0.1493E+03	0.4768E-04	-0.1548E+02
14	-0.	-0.7947E-03	-0.	-0.1493E+03	0.4959E-04	0.1548E+02
15	-0.	-0.7946E-03	-0.	0.1493E+03	0.1907E-04	-0.1548E+02
16	-0.	-0.7946E-03	-0.	0.1493E+03	-0.7629E-05	0.1548E+02
17	0.	-0.8422E-03	-0.	-0.7400E+02	0.3433E-04	-0.5794E+01
18	-0.	-0.8422E-03	-0.	-0.7400E+02	0.3052E-04	0.5794E+01
19	-0.	-0.8422E-03	-0.	0.7406E+02	0.1526E-04	-0.5792E+01
20	-0.	-0.8422E-03	-0.	0.7406E+02	0.7629E-05	0.5792E+01
21	0.	-0.8699E-03	-0.	-0.5370E+02	0.6104E-04	-0.3568E+01
22	-0.	-0.8699E-03	-0.	-0.5370E+02	0.3052E-04	0.3568E+01
23	-0.	-0.8699E-03	-0.	0.5353E+02	0.2289E-04	-0.3571E+01
24	-0.	-0.8699E-03	-0.	0.5353E+02	0.7629E-05	0.3571E+01
25	0.	-0.9008E-03	-0.	-0.5772E+02	0.6104E-04	-0.3409E+01
26	-0.	-0.9008E-03	-0.	-0.5772E+02	0.4578E-04	0.3409E+01
27	-0.	-0.9007E-03	-0.	0.5770E+02	0.7629E-05	-0.3410E+01
28	-0.	-0.9007E-03	-0.	0.5770E+02	-0.1526E-04	0.3410E+01
29	0.	-0.9352E-03	-0.	-0.3011E+02	-0.1965E+02	-0.1697E+01
30	-0.	-0.9352E-03	-0.	-0.3011E+02	-0.1965E+02	0.1697E+01
31	-0.	-0.9352E-03	-0.	0.3016E+02	-0.1962E+02	-0.1696E+01
32	-0.	-0.9352E-03	-0.	0.3016E+02	-0.1962E+02	0.1696E+01

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	0.	-0.62350977E-03	-0.
2	-0.	-0.62350979E-03	-0.
3	0.32632647E-04	-0.62266822E-03	-0.
4	0.32632648E-04	-0.62266825E-03	-0.
5	0.	-0.65682891E-03	-0.
6	-0.	-0.65682890E-03	-0.
7	0.34373992E-04	-0.65589509E-03	-0.
8	0.34373993E-04	-0.65589509E-03	-0.
9	0.	-0.70986806E-03	-0.
10	-0.	-0.70986807E-03	-0.
11	0.37155131E-04	-0.70896238E-03	-0.
12	0.37155132E-04	-0.70896241E-03	-0.
13	0.	-0.79470327E-03	-0.
14	-0.	-0.79470326E-03	-0.
15	0.41588676E-04	-0.79355949E-03	-0.
16	0.41588675E-04	-0.79355948E-03	-0.
17	0.	-0.84220462E-03	-0.
18	-0.	-0.84220464E-03	-0.
19	0.44079506E-04	-0.84108739E-03	-0.
20	0.44079510E-04	-0.84108747E-03	-0.
21	0.	-0.86992865E-03	-0.
22	-0.	-0.86992867E-03	-0.
23	0.45528708E-04	-0.86873981E-03	-0.
24	0.45528712E-04	-0.86873988E-03	-0.
25	0.	-0.90075396E-03	-0.
26	-0.	-0.90075406E-03	-0.
27	0.47139680E-04	-0.89947900E-03	-0.
28	0.47139683E-04	-0.89947906E-03	-0.
29	0.	-0.93520511E-03	-0.
30	-0.	-0.93520521E-03	-0.
31	0.48943195E-04	-0.93389212E-03	-0.
32	0.48943195E-04	-0.93389213E-03	-0.

```
ELEMENT NUMBER          FACE NODE NUMBERS          X,Y AND Z COORDINATES
   X-STRESS          Y-STRESS          Z-STRESS          XY-STRESS       YZ-STRESS       XZ-STRESS
      1                2    4    8      6               0.049075       0.126275       0.100000
       0.000342        -0.000133        -0.000000        0.000025      -0.000000       -0.000000
   10707.002197      -685.049873      2505.487427      299.620590      -0.000382       -0.000034
      1                1    3    7      5               0.049075       0.126275       0.
       0.000342        -0.000133         0.000000        0.000025      -0.000000       -0.000000
   10707.001221      -685.054726      2505.487213      299.620773      -0.000404       -0.000033
      1                2    4    1      3               0.052350       0.001350       0.050000
       0.000310        -0.000121         0.000000        0.000023      -0.000000       -0.000000
    9708.801270      -639.647270      2267.288513      270.055374      -0.000382       -0.000008
      1                5    8    6      7               0.045800       0.251200       0.050000
       0.000373        -0.000145        -0.000000        0.000027      -0.000000       -0.000000
   11705.202026      -730.457397      2743.686157      329.186008      -0.000437       -0.000029
      1                1    2    5      6               0.             0.125000       0.050000
       0.000342        -0.000133        -0.000000        0.000023      -0.000000        0.000000
   10708.900879      -689.397717      2504.950775      281.588280      -0.000415        0.000005
      1                8    4    7      3               0.098150       0.127550       0.050000
       0.000342        -0.000133        -0.000000        0.000026      -0.000000       -0.000000
   10705.102295      -681.036897      2506.023834      317.653118      -0.000366       -0.000018

AVERAGE STRESS FOR ELEMENT  1
   10707.001831      -685.052299      2505.487335      299.620708      -0.000393       -0.000023

      2                6    8   12     10               0.042525       0.376125       0.100000
       0.000421        -0.000212         0.000000        0.000033      -0.000000       -0.000000
   12623.665803     -2576.274384      2511.849640      396.679150      -0.000306       -0.000039
      2                5    7   11      9               0.042525       0.376125       0.
       0.000421        -0.000212        -0.000000        0.000033      -0.000000        0.000000
   12623.671021     -2576.270142      2511.849457      396.680725      -0.000327        0.000007
      2                6    8    5      7               0.045800       0.251200       0.050000
       0.000373        -0.000190        -0.000000        0.000029      -0.000000       -0.000000
   11129.464844     -2378.435608      2187.757324      348.865864      -0.000393       -0.000004
      2                9   12   10     11               0.039250       0.501050       0.050000
       0.000470        -0.000234         0.000000        0.000037      -0.000000       -0.000000
   14117.575977     -2774.108734      2835.941833      444.494114      -0.000065       -0.000021
      2                5    6    9     10               0.             0.375000       0.050000
       0.000421        -0.000212        -0.000000        0.000027       0.000000       -0.000000
   12628.269043     -2573.460327      2513.702179      328.831314       0.000349       -0.000012
      2               12    8   11      7               0.085050       0.377250       0.050000
       0.000421        -0.000212         0.000000        0.000039      -0.000000       -0.000000
   12619.071777     -2579.084198      2509.996887      464.528675      -0.000833       -0.000031

AVERAGE STRESS FOR ELEMENT  2
   12623.670532     -2576.272186      2511.849579      396.680004      -0.000262        0.000006

      3               10   12   16     14               0.035975       0.625950       0.100000
       0.000549        -0.000339        -0.000000        0.000047      -0.000000        0.000000
   15702.379395     -5601.780273      2525.149323      558.466988      -0.000895        0.000009
      3                9   11   15     13               0.035975       0.625950       0.
       0.000549        -0.000339         0.000000        0.000047      -0.000000        0.000000
   15702.378540     -5601.784119      2525.149139      558.469444      -0.000851        0.000021
      3               10   12    9     11               0.039250       0.501050       0.050000
       0.000468        -0.000298         0.000000        0.000041      -0.000000        0.000000
   13286.572266     -5092.158691      2048.603363      487.481991      -0.000808        0.000035
      3               13   16   14     15               0.032700       0.750850       0.050000
       0.000630        -0.000380        -0.000000        0.000052      -0.000000       -0.000000
```

```
  18118.166035    -6111.405579    3001.695221     629.454689      -0.000873    -0.000015
     3               9   10   13   14           0.            0.625000    0.050000
      0.000549       -0.000339      -0.000000       0.000034      -0.            -0.000000
  15708.303345    -5616.644165    2522.914795     411.530926      -0.            -0.000042
     3              16   12   15   11           0.071950      0.626900    0.050000
      0.000549       -0.000338       0.000000       0.000059      -0.000000      0.000000
  15696.455200    -5586.920105    2527.383789     705.405685      -0.001592      0.000126

AVERAGE STRESS FOR ELEMENT  3
  15702.379028    -5601.782288    2525.149231     558.468216      -0.000873    -0.000012

     4              14   16   20   18           0.031375      0.800825    0.100000
      0.000682       -0.000475       0.000000       0.000061      -0.000000      0.000000
  18873.821533    -8898.215454    2493.904633     732.789017      -0.003110      0.000139
     4              13   15   19   17           0.031375      0.800825    0.
      0.000682       -0.000475      -0.000000       0.000061      -0.000000      0.000000
  18873.826416    -8898.194702    2493.904907     732.794998      -0.003110      0.000097
     4              14   16   13   15           0.032700      0.750850    0.050000
      0.000634       -0.000448      -0.000000       0.000057      -0.000000      0.000000
  17443.214111    -8523.259277    2229.988739     679.267792      -0.002816      0.000054
     4              17   20   18   19           0.030050      0.850800    0.050000
      0.000731       -0.000501       0.000000       0.000066      -0.000000      0.000000
  20304.433838    -9273.151001    2757.820740     786.316223      -0.003361      0.000192
     4              13   14   17   18           0.            0.800000    0.050000
      0.000683       -0.000475       0.000000       0.000041      -0.000000     -0.000000
  18890.074951    -8857.601318    2498.118469     496.495834      -0.001135     -0.000015
     4              20   16   19   15           0.062750      0.801650    0.050000
      0.000682       -0.000475       0.000000       0.000081      -0.000000      0.000000
  18857.572998    -8898.808960    2489.691040     969.088127      -0.005051      0.000240

AVERAGE STRESS FOR ELEMENT  4
  18873.824219    -8898.205078    2493.904877     732.792007      -0.003132      0.000107

     5              18   20   24   22           0.029425      0.875775    0.100000
      0.000761       -0.000554      -0.000000       0.000068      -0.000000      0.000000
  20740.573975   -10800.238281    2485.083618     812.704529      -0.006712      0.000202
     5              17   19   23   21           0.029425      0.875775    0.
      0.000761       -0.000554       0.000000       0.000068      -0.000000      0.000000
  20740.571045   -10800.240112    2485.083069     812.716507      -0.006647      0.000259
     5              18   20   17   19           0.030050      0.850800    0.050000
      0.000732       -0.000539       0.000000       0.000066      -0.000000      0.000000
  19905.519141   -10570.723633    2333.723816     791.078819      -0.006570      0.000214
     5              21   24   22   23           0.028800      0.900750    0.050000
      0.000789       -0.000569      -0.000000       0.000070      -0.000000      0.000000
  21575.525635   -11029.754761    2636.442780     834.342148      -0.006778      0.000242
     5              17   18   21   22           0.            0.875000    0.050000
      0.000761       -0.000554      -0.000000       0.000045      -0.000000      0.000000
  20746.919434   -10821.975488    2481.261017     542.170074      -0.003536      0.000011
     5              24   20   23   19           0.058850      0.876550    0.050000
      0.000760       -0.000553       0.000000       0.000090      -0.000000      0.000000
  20734.224854   -10778.603516    2488.905334    1083.250854      -0.009766      0.000439

AVERAGE STRESS FOR ELEMENT  5
  20740.572510   -10800.229136    2485.083313     812.710442      -0.006690      0.000225

     6              22   24   28   26           0.028150      0.925725    0.100000
      0.000822       -0.000615       0.000000       0.000075      -0.000000      0.000000
  22217.501221   -12286.124268    2482.846863     903.242790      -0.008349      0.000274
     6              21   23   27   25           0.028150      0.925725    0.
      0.000822       -0.000615      -0.000000       0.000075      -0.000000      0.000000
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 22217.503174  -12286.106934    2482.846497     903.244225     -0.008316     0.000207
   6              22   24   21   23            0.028800      0.930750     0.050000
    0.000789      -0.000596      -0.000000       0.000073     -0.000000     0.000000
 21246.155668  -12002.301880    2310.964508     877.104935     -0.008196     0.000318
   6              25   28   26   27            0.027500      0.950700     0.050000
    0.000856      -0.000634       0.000000       0.000077     -0.000000     0.000000
 23188.844238  -12569.930054    2654.728516     929.382034     -0.008458     0.000185
   6              21   22   25   26            0.            0.925000     0.050000
    0.000823      -0.000616       0.000000       0.000049     -0.000000    -0.000000
 22227.092285  -12313.166260    2478.481506     583.293869     -0.008033    -0.000003
   6              28   24   27   23            0.056300      0.926450     0.050000
    0.000822      -0.000614      -0.000000       0.000102     -0.000000     0.000000
 22207.911377  -12259.065674    2487.211517    1223.193054     -0.008599     0.000484

AVERAGE STRESS FOR ELEMENT  6
 22217.502197  -12286.115723    2482.846649     903.243484     -0.008327     0.000241

   7              26   28   32   30            0.026825      0.975700     0.100000
    0.000895      -0.000688      -0.000000       0.000084     -0.000000     0.000000
 23957.568359  -14017.410889    2485.036987    1011.101074     -0.009102     0.000151
   7              25   27   31   29            0.026825      0.975700     0.
    0.000895      -0.000688       0.000000       0.000084     -0.000000     0.000000
 23957.563232  -14017.426270    2485.036682    1011.088852     -0.009146     0.000214
   7              26   28   25   27            0.027500      0.950700     0.050000
    0.000855      -0.000665       0.000000       0.000080     -0.000000     0.000000
 22818.572998  -13666.146606    2288.106506     955.935951     -0.008829     0.000146
   7              29   32   30   31            0.026150      1.000700     0.050000
    0.000934      -0.000710      -0.000000       0.000089     -0.000000     0.000000
 25096.558594  -14368.690796    2681.967133    1066.254105     -0.009430     0.000192
   7              25   26   29   30            0.            0.975000     0.050000
    0.000856      -0.000689       0.000000       0.000053     -0.000000     0.000000
 23995.057129  -14044.225586    2487.707886     633.359451     -0.012464     0.000080
   7              32   28   31   27            0.053650      0.976400     0.050000
    0.000893      -0.000686      -0.000000       0.000116     -0.000000     0.000000
 23920.074219  -13990.611694    2482.365692    1388.830429     -0.005820     0.000318

AVERAGE STRESS FOR ELEMENT  7
 23957.566162  -14017.418579    2485.036865    1011.095024     -0.009124     0.000172
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YIELD CHECK AFTER 1 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.000340	0.	10192.2	30000.0	30000000.0	0.2500
2	0.000447	0.	13418.6	30000.0	30000000.0	0.2500
3	0.000622	0.	18647.1	30000.0	30000000.0	0.2500
4	0.000807	0.	24213.5	30000.0	30000000.0	0.2500
5	0.000915	0.	27464.1	30000.0	30000000.0	0.2500
6	0.001001	0.000001	30000.0	30000.0	29975253.0	0.2502
7	0.001101	0.000101	30000.3	30000.0	27250280.7	0.2729

YIELD CHECK AFTER 2 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.000343	0.	10289.5	30000.0	30000000.0	0.2500
2	0.000452	0.	13546.8	30000.0	30000000.0	0.2500
3	0.000627	0.	18825.1	30000.0	30000000.0	0.2500
4	0.000815	0.	24444.7	30000.0	30000000.0	0.2500
5	0.000924	0.	27726.2	30000.0	30000000.0	0.2500
6	0.001011	0.000011	30000.0	30000.0	29687895.0	0.2526
7	0.001129	0.000129	30000.4	30000.0	26566693.5	0.2786

YIELD CHECK AFTER 3 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.000344	0.	10323.0	30000.0	30000000.0	0.2500
2	0.000453	0.	13590.8	30000.0	30000000.0	0.2500
3	0.000630	0.	18886.3	30000.0	30000000.0	0.2500
4	0.000817	0.	24524.2	30000.0	30000000.0	0.2500
5	0.000927	0.	27816.4	30000.0	30000000.0	0.2500
6	0.001015	0.000015	30000.0	30000.0	29545952.7	0.2538
7	0.001138	0.000138	30000.4	30000.0	26369190.5	0.2803

YIELD CHECK AFTER 4 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.000344	0.	10334.5	30000.0	30000000.0	0.2500
2	0.000454	0.	13606.0	30000.0	30000000.0	0.2500
3	0.000630	0.	18907.5	30000.0	30000000.0	0.2500
4	0.000818	0.	24551.7	30000.0	30000000.0	0.2500
5	0.000928	0.	27847.6	30000.0	30000000.0	0.2500
6	0.001017	0.000017	30000.1	30000.0	29490130.7	0.2542
7	0.001140	0.000140	30000.4	30000.0	26307153.7	0.2808

YIELD CHECK AFTER 5 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.000345	0.	10338.5	30000.0	30000000.0	0.2500
2	0.000454	0.	13611.3	30000.0	30000000.0	0.2500
3	0.000630	0.	18914.7	30000.0	30000000.0	0.2500
4	0.000819	0.	24561.1	30000.0	30000000.0	0.2500
5	0.000929	0.	27858.3	30000.0	30000000.0	0.2500
6	0.001018	0.000018	30000.1	30000.0	29469845.7	0.2544
7	0.001141	0.000141	30000.4	30000.0	26286815.2	0.2809

NODE	X-DISPL	Y-DISPL	Z-DISPL	X-FORCE	Y-FORCE	Z-FORCE
1	0.	-0.6325E-03	-0.	-0.6583E+02	0.1144E-04	-0.1541E+02
2	-0.	-0.6325E-03	-0.	-0.6583E+02	0.1144E-04	0.1541E+02
3	-0.	-0.6326E-03	-0.	0.6581E+02	0.5722E-05	-0.1541E+02
4	-0.	-0.6326E-03	-0.	0.6581E+02	-0.	0.1541E+02
5	0.	-0.6663E-03	-0.	-0.1470E+03	0.3052E-04	-0.2898E+02
6	-0.	-0.6663E-03	-0.	-0.1470E+03	0.2480E-04	0.2898E+02
7	-0.	-0.6663E-03	-0.	0.1470E+03	-0.9537E-05	-0.2897E+02
8	-0.	-0.6663E-03	-0.	0.1470E+03	-0.9537E-05	0.2897E+02
9	0.	-0.7202E-03	-0.	-0.1778E+03	0.2861E-04	-0.2481E+02
10	-0.	-0.7202E-03	-0.	-0.1778E+03	0.2289E-04	0.2481E+02
11	-0.	-0.7202E-03	-0.	0.1778E+03	-0.1335E-04	-0.2481E+02
12	-0.	-0.7202E-03	-0.	0.1778E+03	-0.1335E-04	0.2481E+02
13	0.	-0.8062E-03	-0.	-0.1515E+03	0.4005E-04	-0.1571E+02
14	-0.	-0.8062E-03	-0.	-0.1515E+03	0.3052E-04	0.1571E+02
15	-0.	-0.8062E-03	-0.	0.1514E+03	0.2289E-04	-0.1571E+02
16	-0.	-0.8062E-03	-0.	0.1514E+03	-0.1144E-04	0.1571E+02
17	0.	-0.8544E-03	-0.	-0.7507E+02	0.4578E-04	-0.5878E+01
18	-0.	-0.8544E-03	-0.	-0.7507E+02	0.6485E-04	0.5878E+01
19	-0.	-0.8544E-03	-0.	0.7513E+02	0.1144E-04	-0.5875E+01
20	-0.	-0.8544E-03	-0.	0.7513E+02	0.3815E-05	0.5875E+01
21	0.	-0.8825E-03	-0.	-0.5387E+02	0.4578E-04	-0.3564E+01
22	-0.	-0.8825E-03	-0.	-0.5387E+02	0.4578E-04	0.3564E+01
23	-0.	-0.8825E-03	-0.	0.5379E+02	0.1526E-04	-0.3567E+01
24	-0.	-0.8825E-03	-0.	0.5379E+02	0.1526E-04	0.3567E+01
25	0.	-0.9143E-03	-0.	-0.5316E+02	0.4578E-04	-0.2905E+01
26	-0.	-0.9143E-03	-0.	-0.5316E+02	0.5341E-04	0.2905E+01
27	-0.	-0.9142E-03	-0.	0.5314E+02	-0.7629E-05	-0.2907E+01
28	-0.	-0.9142E-03	-0.	0.5314E+02	0.7629E-05	0.2907E+01
29	0.	-0.9531E-03	-0.	-0.2570E+02	-0.1965E+02	-0.1231E+01
30	-0.	-0.9531E-03	-0.	-0.2570E+02	-0.1965E+02	0.1231E+01
31	-0.	-0.9530E-03	-0.	0.2575E+02	-0.1962E+02	-0.1230E+01
32	-0.	-0.9530E-03	-0.	0.2575E+02	-0.1962E+02	0.1230E+01

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	0.	-0.63254354E-03	-0.
2	-0.	-0.63254354E-03	-0.
3	0.33105447E-04	-0.63168979E-03	-0.
4	0.33105448E-04	-0.63168979E-03	-0.
5	0.	-0.66634539E-03	-0.
6	-0.	-0.66634540E-03	-0.
7	0.34872022E-04	-0.66539806E-03	-0.
8	0.34872022E-04	-0.66539805E-03	-0.
9	0.	-0.72015302E-03	-0.
10	-0.	-0.72015302E-03	-0.
11	0.37693455E-04	-0.71923422E-03	-0.
12	0.37693456E-04	-0.71923424E-03	-0.
13	0.	-0.80621735E-03	-0.
14	-0.	-0.80621735E-03	-0.
15	0.42191235E-04	-0.80505700E-03	-0.
16	0.42191235E-04	-0.80505700E-03	-0.
17	0.	-0.85440694E-03	-0.
18	-0.	-0.85440695E-03	-0.
19	0.44718155E-04	-0.85327352E-03	-0.
20	0.44718157E-04	-0.85327357E-03	-0.
21	0.	-0.88253275E-03	-0.
22	-0.	-0.88253273E-03	-0.
23	0.46188351E-04	-0.88132654E-03	-0.
24	0.46188351E-04	-0.88132655E-03	-0.
25	0.	-0.91428887E-03	-0.
26	-0.	-0.91428887E-03	-0.
27	0.47847790E-04	-0.91299054E-03	-0.
28	0.47847790E-04	-0.91299054E-03	-0.
29	0.	-0.95308998E-03	-0.
30	-0.	-0.95309000E-03	-0.
31	0.49878437E-04	-0.95173762E-03	-0.
32	0.49878440E-04	-0.95173766E-03	-0.

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ELEMENT NUMBER          FACE NODE NUMBERS        X,Y AND Z COORDINATES
    X-STRESS            Y-STRESS         Z-STRESS        XY-STRESS       YZ-STRESS       XZ-STRESS
      1                  2    4    8     6              0.049075      0.126275      0.100000
        0.000347        -0.000135         0.000000         0.000025       -0.000000       -0.000000
    10862.130859      -694.976860      2541.788483       303.962002       -0.000196       -0.000049
      1                  1    3    7     5              0.049075      0.126275      0.
        0.000347        -0.000135         0.000000         0.000025       -0.000000       -0.000000
    10862.130859      -694.977531      2541.788300       303.961662       -0.000207       -0.000026
      1                  2    4    1     3              0.052350      0.001350      0.050000
        0.000315        -0.000123         0.000000         0.000023       -0.000000       -0.000000
     9849.467896      -648.914421      2300.138367       273.968147       -0.000196       -0.000007
      1                  5    8    6     7              0.045800      0.251200      0.050000
        0.000379        -0.000147        -0.000000         0.000028       -0.000000       -0.000000
    11874.793579      -741.040161      2783.438354       333.955517       -0.000218       -0.000062
      1                  1    2    5     6              0.            0.125000      0.050000
        0.000347        -0.000135        -0.000000         0.000024       -0.000000       -0.000000
    10864.057495      -699.079773      2541.244446       285.668125       -0.000437       -0.000006
      1                  8    4    7     3              0.098150      0.127550      0.050000
        0.000347        -0.000135        -0.000000         0.000027        0.000000       -0.000000
    10860.203979      -690.874695      2542.332306       322.255535        0.000086       -0.000058

AVERAGE STRESS FOR ELEMENT  1
    10862.130859      -694.977226      2541.788422       303.961849       -0.000196       -0.000037

      2                  6    8   12    10              0.042525      0.376125      0.100000
        0.000427        -0.000215         0.000000         0.000034       -0.000000       -0.000000
    12806.568848     -2613.598595      2548.242737       402.426529       -0.000458       -0.000039
      2                  5    7   11     9              0.042525      0.376125      0.
        0.000427        -0.000215        -0.000000         0.000034       -0.000000        0.000000
    12806.569458     -2613.598114      2548.242584       402.426609       -0.000480        0.000019
      2                  6    8    5     7              0.045800      0.251200      0.050000
        0.000278        -0.000193        -0.000000         0.000029       -0.000000       -0.000000
    11290.714600     -2412.855569      2219.454742       353.919769       -0.000568       -0.000070
      2                  9   12   10    11              0.039250      0.501050      0.050000
        0.000477        -0.000237         0.000000         0.000038       -0.000000       -0.000000
    14322.423584     -2814.301483      2877.030518       450.933422       -0.000393       -0.000021
      2                  5    6    9    10              0.            0.375000      0.050000
        0.000428        -0.000[illegible]        -0.000000         0.000028       -0.000000        0.000000
    12811.234497     -2610.74[illegible]      2550.122040       333.594875       -0.000480        0.000019
      2                 12    [illegible]   11     7              0.085050      0.377250      0.050000
        0.000427        -0.000215         0.000000         0.000039       -0.000000       -0.000000
    12801.903564     -2616.450653      2546.363190       471.258339       -0.000403       -0.000039

AVERAGE STRESS FOR ELEMENT  2
    12806.569214     -2613.598602      2548.242645       402.426586       -0.000415       -0.000005

      3                 10   12   16    14              0.035975      0.625950      0.100000
        0.000557        -0.000344        -0.000000         0.000047       -0.000000        0.000000
    15929.484766     -5682.942261      2561.735535       566.559029       -0.000917        0.000043
      3                  9   11   15    13              0.035975      0.625950      0.
        0.000557        -0.000344         0.000000         0.000047       -0.000000        0.000000
    15929.884033     -5682.943542      2561.735199       566.560333       -0.000895        0.000033
      3                 10   12    9    11              0.039250      0.501050      0.050000
        0.000475        -0.000302         0.000000         0.000041       -0.000000        0.000000
    13479.075562     -5165.925913      2078.284882       494.544838       -0.000851        0.000053
      3                 13   16   14    15              0.032700      0.750850      0.050000
        0.000639        -0.000385        -0.000000         0.000053       -0.000000        0.000000
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  18390.653115    -6199.949768    3045.185883     638.574547      -0.000873          0.000014
     3               9   10   13    14             0.             0.625000    0.050000
      0.000557       -0.000344       -0.000000        0.000035      -0.000000         -0.000000
  15935.894409    -5698.019897    2559.468658     417.493385      -0.000480         -0.000003
     3              16   12   15    11             0.071950       0.626900    0.050000
      0.000557       -0.000343        0.000000        0.000063      -0.000000          0.000000
  15923.874390    -5667.865540    2564.002258     715.625931      -0.001166          0.000133

AVERAGE STRESS FOR ELEMENT  3
  15929.884277    -5682.943054    2561.735321     566.559593      -0.000895          0.000022

     4              14   16   20    18             0.031375       0.800825    0.100000
      0.000692       -0.000482        0.000000        0.000062      -0.000000          0.000000
  19147.277344    -9027.131592    2530.037689     743.408600      -0.002248          0.000126
     4              13   15   19    17             0.031375       0.800825    0.
      0.000692       -0.000482       -0.000000        0.000062      -0.000000          0.000000
  19147.279297    -9027.122559    2530.037994     743.412033      -0.002259          0.000095
     4              14   16   13    15             0.032700       0.750850    0.050000
      0.000643       -0.000455       -0.000000        0.000057      -0.000000          0.000000
  17695.941162    -8646.748779    2262.298187     689.110550      -0.002008          0.000061
     4              17   20   18    19             0.030050       0.850800    0.050000
      0.000742       -0.000539        0.000000        0.000066      -0.000000          0.000000
  20598.615479    -9407.505249    2797.777466     797.710068      -0.002510          0.000154
     4              13   14   17    18             0.             0.800000    0.050000
      0.000693       -0.000492        0.000000        0.000042      -0.000000          0.000000
  19163.765625    -9026.516602    2534.312195     503.690582      -0.001179          0.000032
     4              20   16   19    15             0.062750       0.801650    0.050000
      0.000692       -0.000481        0.000000        0.000082      -0.000000          0.000000
  19130.791504    -9027.737305    2525.763489     983.129929      -0.003332          0.000178

AVERAGE STRESS FOR ELEMENT  4
  19147.278564    -9027.127197    2530.037811     743.410263      -0.002270          0.000105

     5              18   20   24    22             0.029425       0.875775    0.100000
      0.000772       -0.000562       -0.000000        0.000069      -0.000000          0.000000
  21041.071777   -10956.717285    2521.085938     824.498390      -0.002161          0.000031
     5              17   19   23    21             0.029425       0.875775    0.
      0.000772       -0.000562        0.000000        0.000069      -0.000000          0.000000
  21041.067383   -10956.734619    2521.085907     824.505501      -0.002074          0.000076
     5              18   20   17    19             0.030050       0.850800    0.050000
      0.000743       -0.000545        0.000000        0.000067      -0.000000          0.000000
  20194.019043   -10723.885010    2367.533508     802.556496      -0.002085          0.000039
     5              21   24   22    23             0.028800       0.900750    0.050000
      0.000801       -0.000578       -0.000000        0.000071      -0.000000          0.000000
  21888.120117   -11189.566895    2674.638275     846.447327      -0.002106          0.000042
     5              17   18   21    22             0.             0.875000    0.050000
      0.000772       -0.000562       -0.000000        0.000046      -0.000000         -0.000000
  21047.509277   -10978.720337    2517.197235     550.042702      -0.000458         -0.000018
     5              24   20   23    19             0.058850       0.876550    0.050000
      0.000771       -0.000561        0.000000        0.000092      -0.000000          0.000000
  21034.630127   -10934.731079    2524.974670    1098.961090      -0.003707          0.000146

AVERAGE STRESS FOR ELEMENT  5
  21041.070068   -10956.725464    2521.086060     824.501816      -0.002128          0.000032

     6              22   24   28    26             0.028150       0.925725    0.100000
      0.000834       -0.000634        0.000000        0.000077      -0.000000          0.000000
  22042.347656   -12452.996094    2439.703918     903.426712      -0.000897          0.000060
     6              21   23   27    25             0.028150       0.925725    0.
      0.000834       -0.000634       -0.000000        0.000077      -0.000000          0.000000
```

22042.348633	-12452.991943	2439.703827	903.429558	-0.000908	0.000048
6	22 24 21	23	0.028800	0.900750	0.050000
0.000800	-0.000614	-0.000000	0.000075	-0.000000	0.000000
21069.486328	-12165.332153	2265.122284	877.064842	-0.000983	0.000034
6	25 28 26	27	0.027500	0.950700	0.050000
0.000868	-0.000654	0.000000	0.000079	-0.000000	0.000000
23016.210205	-12740.655273	2614.285583	929.791481	-0.000865	0.000009
6	21 22 25	26	0.	0.925000	0.050000
0.000835	-0.000635	0.000000	0.000049	-0.000000	-0.000000
22052.231201	-12481.508789	2434.963501	579.173492	-0.000192	-0.000046
6	28 24 27	23	0.056300	0.926450	0.050000
0.000834	-0.000633	-0.000000	0.000105	-0.000000	0.000000
22032.464844	-12424.479370	2444.444000	1227.682938	-0.001553	0.000047
AVERAGE STRESS FOR ELEMENT 6					
22042.347900	-12452.994019	2439.703766	903.428131	-0.000876	0.000070
7	26 28 32	30	0.026825	0.975700	0.100000
0.000910	-0.000774	0.000000	0.000090	-0.000000	0.000000
20458.243408	-14105.928223	1784.642517	921.307014	-0.002884	0.000039
7	25 27 31	29	0.026825	0.975700	0.
0.000910	-0.000774	-0.000000	0.000090	-0.000000	0.000000
20458.246094	-14105.913940	1784.641800	921.310631	-0.002902	0.000111
7	26 28 25	27	0.027500	0.950700	0.050000
0.000868	-0.000748	-0.000000	0.000085	-0.000000	0.000000
19404.798096	-13752.305298	1588.029526	870.640503	-0.002772	0.000005
7	29 32 30	31	0.026150	1.000700	0.050000
0.000952	-0.000801	0.000000	0.000095	-0.000000	0.000000
21511.690918	-14459.537109	1581.254639	971.977127	-0.003052	0.000125
7	25 26 29	30	0.	0.975000	0.050000
0.000912	-0.000776	-0.000000	0.000053	-0.000000	-0.000000
20493.374512	-14136.221680	1785.999008	541.278091	-0.001848	-0.000026
7	32 28 31	27	0.053650	0.976400	0.050000
0.000908	-0.000773	0.000000	0.000127	-0.000000	0.000000
20423.114258	-14075.620483	1783.285278	1301.339523	-0.003983	0.000232
AVERAGE STRESS FOR ELEMENT 7					
20459.244385	-14105.921143	1784.642197	921.308861	-0.002912	0.000112

YIELD CHECK AFTER 6 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.000345	0.	10339.9	30000.0	30000000.0	0.2500
2	0.000454	0.	13613.1	30000.0	30000000.0	0.2500
3	0.000631	0.	18917.2	30000.0	30000000.0	0.2500
4	0.000819	0.	24564.3	30000.0	30000000.0	0.2500
5	0.000929	0.	27862.0	30000.0	30000000.0	0.2500
6	0.001018	0.000018	30000.1	30000.0	29462683.7	0.2545
7	0.001142	0.000142	30000.4	30000.0	26279976.7	0.2810

APPENDIX E--HEATED ELEMENT CYCLING EXAMPLE-INPUT AND OUTPUT DATA

TITLE	PROJECT NUMBER	ANALYST	SHEET
Heated Element Cycling Example	4880	Ibrahim	SHEET 1 OF 1

FORTRAN STATEMENT

```
$DATA
    1 8   1   P  1    3   0   2   6   0   0
          0.              1.0000          -0.
          0.              1.0000          -1.0000
          1.0000          1.0000          0.
          1.0000          1.0000          -1.0000
          0.              -0.             -0.
          0.              -0.             -1.0000
          1.0000          -0.             0.
          1.0000          -0.             -1.0000
    8
    1 1   1   8
   17650000.0000          0.3300          9.8000
      5600.0000           0.              4.0500          -0.
    2 4   8   6  1    3   7   5   1 -200.000
    1
    1 0   1   0      -0.         -0.          -0.        -0.
    2 0   1   1      -0.         -0.          -0.        -0.
    3 0   1   1      -0.         -0.          -0.        -0.
    4 0   1   1      -0.         -0.          -0.        -0.
    5 0   0   0      -0.         -0.          -0.        -0.
    6 0   1   1      -0.         -0.          -0.        -0.
    7 0   0   1      -0.         -0.          -0.        -0.
    8 0   1   1      -0.         -0.          -0.        -0.
```

NASA-C-836 (REV. 9-14-59)

```
UNUSED CORE                          60455 THRU  64673

  BEGIN EXECUTION.
  1   8   1   8   1   3   0   2   6   0   0
  1           C.                 1.0000         -0.
  2           C.                 1.0000         -1.0000
  3           1.0000             1.0000          0.
  4           1.0000             1.0000         -1.0000
  5           C.               -0.               0.
  6           C.               -C.              -1.0000
  7           1.0000           -0.               0.
  8           1.0000           -0.              -1.0000
  1   1   1   8
  1        56CC.0000             0.              4.0500            -0.
  1   176500CC.0000              C.3300          9.8000
  2   4   8   6   1   3   7   5   1  -200.000
  1   0   1   0     -0.             -C.             -0.             -0.
  2   0   1   1     -0.             -0.             -0.             -0.
  3   0   1   1     -C.             -0.             -0.             -0.
  4   0   1   1     -C.             -0.             -0.             -0.
  5   0   0   0     -0.             -0.             -0.             -0.
  6   0   1   1     -C.             -0.             -0.             -0.
  7   0   0   1     -0.             -0.             -0.             -0.
  8   0   1   1     -C.             -0.             -0.             -0.
```

VOR 5446 PRICE
ROTATED SYSTEM

NODE	X-DISPL	Y-DISPL	Z-DISPL	X-FORCE	Y-FORCE	Z-FORCE
1	0.	-0.2607E-02	-0.	-0.8649E+04	0.3174E-02	0.2441E-03
2	-0.	-0.2607E-02	0.2607E-02	-0.8648E+04	0.2930E-02	-0.2686E-02
3	-0.	-0.2607E-02	0.1153E-08	0.8649E+04	0.3418E-02	0.3418E-02
4	-0.	-0.2607E-02	0.2607E-02	0.8648E+04	0.2441E-02	-0.3418E-02
5	-0.	-0.	-0.	-0.8649E+04	-0.5127E-02	0.4883E-02
6	-0.	0.2212E-08	0.2607E-02	-0.8648E+04	-0.1465E-02	-0.1465E-02
7	-0.	-0.	-0.1281E-08	0.8649E+04	-0.4395E-02	0.2197E-02
8	-0.	0.2328E-08	0.2607E-02	0.8649E+04	-0.1465E-02	-0.1709E-02

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	0.	-0.26067588E-C2	-0.
2	-0.	-0.26067581E-C2	C.26068006E-02
3	-0.	-0.26067992E-C2	C.11932570E-08
4	-0.	-0.26067578E-C2	C.26C68004E-02
5	-0.	0.	-0.
6	-0.	0.22118911E-C8	C.26067990E-02
7	-0.	0.	-0.12805685E-08
8	-0.	0.23283064E-08	C.26067990E-02

```
ELEMENT NUMBER        FACE NODE NUMBERS        X,Y AND Z COORDINATES
   X-STRESS          Y-STRESS          Z-STRESS          XY-STRESS          YZ-STRESS          XZ-STRESS
      1               2    4    8    6              0.500000       0.500000      -1.000000
       0.001960        -0.000647         -0.000647          0.000000          -0.000000          -0.000000
   34594.003906         0.003906          0.011230          0.000966          -0.001448          -0.000386
      1               1    3    7    5              0.500000       0.500000      -0.
       0.001960        -0.000647         -0.000647         -0.000000          -0.000000          -0.000000
   34594.014648         0.030640          0.017578         -0.001443          -0.000917          -0.000290
      1               2    4    1    3              0.500000       1.000000      -0.500000
       0.001960        -0.000647         -0.000647         -0.000000          -0.000000           0.000000
   34594.006736         0.014526          0.010742         -0.000336          -0.001400           0.003476
      1               5    8    6    7              0.500000      -0.            -0.500000
       0.001960        -0.000647         -0.000647         -0.000000          -0.000000          -0.000000
   34594.011230         0.020020          0.017822         -0.000064          -0.000724          -0.004055
      1               1    2    5    6              0.             0.500000      -0.500000
       0.001960        -0.000647         -0.000647         -0.000000          -0.000000           0.000000
   34594.009766         0.018555          0.014893         -0.000354          -0.004055           0.000145
      1               8    4    7    3              1.000000       0.500000      -0.500000
       0.001960        -0.000647         -0.000647         -0.000000           0.000000          -0.000000
   34594.008789         0.016602          0.013672         -0.000046           0.001254          -0.000917

AVERAGE STRESS FOR ELEMENT  1
   34594.010254         0.018555          0.016113         -0.000297          -0.001062          -0.000241
```

YIELC CHECK AFTER 1 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.001960	0.001636	5717.4	5600.0	2917054.5	0.4719

NODE	X-DISPL	Y-DISPL	Z-DISPL	X-FORCE	Y-FORCE	Z-FORCE
1	-0.	-0.2885E-02	C.	-0.1429E+04	-0.2441E-02	-0.5371E-C2
2	0.	-0.2885E-02	0.2885E-02	-0.1429E+04	-0.2197E-02	0.2197E-02
3	0.	-0.2885E-02	0.6286E-08	0.1429E+04	-0.2197E-02	-0.3174E-02
4	0.	-C.2885E-02	C.2885E-02	0.1429E+04	-0.4395E-02	0.4150E-02
5	0.	C.	0.	-0.1429E+04	0.1221E-02	-0.2686E-02
6	0.	C.5588E-08	C.2885E-C2	-0.1429E+04	0.3174E-02	0.2930E-02
7	0.	0.	-C.1863E-08	0.1429E+04	0.9766E-03	-0.4395E-02
8	0.	C.1304E-07	C.2885E-02	0.1429[illegible]+04	0.6348E-02	0.5127E-02

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	-0.	-0.28849259E-02	0.
2	0.	-0.28849253E-02	0.28849335E-02
3	0.	-0.28849281E-02	0.62864274E-08
4	0.	-0.28849253E-02	0.28849375E-02
5	0.	-0.	0.
6	0.	0.55879354E-08	0.28849314E-02
7	0.	-0.	-0.18626451E-08
8	0.	0.13038516E-07	0.28849319E-02

```
ELEMENT NUMBER       FACE NODE NUMBERS       X,Y AND Z COORDINATES
    X-STRESS          Y-STRESS         Z-STRESS        XY-STRESS       YZ-STRESS      XZ-STRESS
       1              2    4    8    6               0.500000     0.500000    -1.000000
        0.001960      -0.000925        -0.000925        0.000000       -0.000000        0.000000
    5717.408447      -0.024902        -0.014648        0.003706       -0.001903        0.002199
       1              1    3    7    5               0.500000     0.500000    -0.
        0.001960      -0.000925        -0.000925       -0.000000       -0.000000        0.000000
    5717.430786       0.012329        -0.004639       -0.001124       -0.000952        0.002185
       1              2    4    1    3               0.500000     1.000000    -0.500000
        0.001960      -0.000925        -0.000925        0.000000       -0.000000        0.000000
    5717.417236      -0.009277        -0.011719        0.001269       -0.001889        0.005052
       1              5    8    6    7               0.500000    -0.          -0.500000
        0.001960      -0.000925        -0.000925        0.000000       -0.000000       -0.000000
    5717.420166      -0.005615        -0.009277        0.001288       -0.000988       -0.000678
       1              1    2    5    6               0.           0.500000    -0.500000
        0.001960      -0.000925        -0.000925        0.000000       -0.000000        0.000000
    5717.419678       0.002686        -0.005371        0.001245       -0.001903        0.002689
       1              8    4    7    3               1.000000     0.500000    -0.500000
        0.001960      -0.000925        -0.000925        0.000000       -0.000000        0.000000
    5717.419434      -0.015381        -0.013916        0.001305       -0.000959        0.001694

AVERAGE STRESS FOR ELEMENT  1
    5717.420166      -0.005859        -0.009033        0.001275       -0.001413        0.002177
```

YIELD CHECK AFTER 2 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.001960	0.001636	5717.4	5600.0	2917053.7	0.4719

ELEMENT NUMBER	EQUIVALENT TOTAL STRAIN (PERCENT)	PLASTIC STRAIN COMPONENTS (PERCENT) X-DIR	Y-DIR	Z-DIR
1	0.19233	0.16361	-0.08180	-0.08180

01 EXIT IN RETSCP

```
UNUSED CORE                     60455 THRU  64673

  BEGIN EXECUTION.
  1   8   1   8   1   3   0   2   6   0   1
  1          0.              1.0000          -0.
  2          0.              1.0000          -1.0000
  3           1.0000         1.0000           0.
  4           1.0000         1.0000          -1.0000
  5          0.             -0.               0.
  6          C.             -0.              -1.0000
  7           1.0000        -0.               0.
  8           1.0000        -0.              -1.0000
  1   1   1   8
  1       56CC.0000          0.               4.0500         -0.
  1   176500CC.0000          0.3300           9.8000
    1     0.16360660    -0.08180325    -0.08180334
  2   4   8   6   1   3   7   5   1   200.000
  1   0   1   0    -0.               -0.               -0.               -0.
  2   0   1   1    -0.               -0.               -0.               -0.
  3   0   1   1    -0.               -0.               -0.               -0.
  4   0   1   1    -0.               -0.               -0.               -0.
  5   0   0   0    -0.               -0.               -0.               -0.
  6   0   1   1    -0.               -0.               -0.               -0.
  7   0   0   1    -0.               -0.               -0.               -0.
  8   0   1   1    -0.               -0.               -0.               -0.
```

ROTATED SYSTEM

NODE	X-DISPL	Y-DISPL	Z-DISPL	X-FORCE	Y-FORCE	Z-FORCE
1	-0.	0.2329E-02	0.	0.1587E+05	-0.5371E-02	-0.1221E-02
2	0.	0.2329E-02	-0.2329E-02	0.1587E+05	-0.3906E-02	0.3418E-02
3	0.	0.2329E-02	-0.1106E-08	-0.1587E+05	-0.4639E-02	-0.3662E-02
4	0.	0.2329E-02	-0.2329E-02	-0.1587E+05	-0.4395E-02	0.2930E-02
5	0.	0.	0.	0.1587E+05	0.5859E-02	-0.4395E-02
6	0.	-0.1513E-08	-0.2329E-02	0.1587E+05	0.4395E-02	0.2441E-02
7	0.	0.	0.1164E-08	-0.1587E+05	0.5127E-02	-0.2197E-02
8	0.	-0.1863E-08	-0.2329E-02	-0.1587E+05	0.4155E-02	0.2441E-02

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	-0.	0.23286680E-C2	0.
2	0.	0.23286676E-C2	-0.23286687E-J2
3	0.	0.23286684E-C2	-C.11059456E-08
4	0.	0.23286670E-02	-0.23286690E-J2
5	0.	-0.	0.
6	0.	-0.15133992E-C8	-0.23286677E-J2
7	0.	-0.	0.116[illegible]1532E-08
8	0.	-0.18626451E-C8	-0.23286673E-02

```
ELEMENT NUMBER        FACE NODE NUMBERS         X,Y AND Z COORDINATES
   X-STRESS          Y-STRESS        Z-STRESS        XY-STRESS       YZ-STRESS       XZ-STRESS
     1                2    4    8    6           0.500000      0.500000     -1.000000
        -0.003596       0.001187        0.001187       -0.000000        0.000000        0.000000
   -63470.573242       -0.015747       -0.012939       -0.002897        0.000338        0.000338
     1                1    3    7    5           0.500000      0.500000     -0.
        -0.003596       0.001187        0.001187        0.000000       -0.000000        0.000000
   -63470.581055       -0.032227       -0.019287        0.001540       -0.000241        0.000290
     1                2    4    1    3           0.500000      1.000000     -0.500000
        -0.003596       0.001187        0.001187       -0.000000        0.000000       -0.000000
   -63470.574707       -0.021118       -0.013428       -0.000581        0.000676       -0.004442
     1                5    8    6    7           0.500000     -0.           -0.500000
        -0.003596       0.001187        0.001187       -0.000000       -0.000000        0.000000
   -63470.579102       -0.026489       -0.019287       -0.000807       -0.000193        0.005021
     1                1    2    5    6           0.            0.500000     -0.500000
        -0.003596       0.001187        0.001187       -0.000000        0.000000       -0.000000
   -63470.576660       -0.024292       -0.015869       -0.000517        0.002704       -0.000048
     1                8    4    7    3           1.000000      0.500000     -0.500000
        -0.003596       0.001187        0.001187       -0.000000       -0.000000        0.000000
   -63470.576172       -0.024170       -0.016113       -0.000871       -0.002558        0.000869

AVERAGE STRESS FOR ELEMENT  1
   -63470.577148       -0.024292       -0.017090       -0.000621        0.000048        0.000338
```

YIELD CHECK AFTER 1 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.003596	0.003266	5834.4	5600.0	1622433.1	0.4844

ROTATED SYSTEM

NODE	X-DISPL	Y-DISPL	Z-DISPL	X-FORCE	Y-FORCE	Z-FORCE
1	0.	0.2864E-02	-0.	0.1459E+04	-0.1147E-01	-0.6348E-02
2	-0.	0.2884E-02	-0.2884E-02	0.1459E+04	-0.7080E-02	0.9033E-02
3	-0.	0.2884E-02	-0.1164E-07	-0.1459E+04	-0.8057E-02	-0.8301E-02
4	-0.	0.2884E-02	-0.2884E-02	-0.1459E+04	-0.1123E-01	0.6836E-02
5	-0.	-0.	-0.	0.1459E+04	0.1221E-01	-0.7813E-02
6	-0.	-0.1490E-07	-0.2884E-02	0.1459E+04	0.6592E-02	0.7080E-02
7	-0.	-0.	0.4657E-08	-0.1459E+04	0.8545E-02	-0.8545E-02
8	-0.	-0.1490E-07	-0.2884E-02	-0.1459E+04	0.1074E-01	0.9521E-02

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	0.	0.28837940E-02	-0.
2	-0.	0.28837938E-02	-0.28838096E-02
3	-0.	0.28838008E-02	-0.11641532E-07
4	-0.	0.28837863E-02	-0.28838180E-02
5	-0.	0.	-0.
6	-0.	-0.14901161E-07	-0.28838082E-02
7	-0.	0.	0.46566129E-08
8	-0.	-0.14901161E-07	-0.28837975E-02

```
ELEMENT NIMBER          FACE NODE NUMBERS          X,Y AND Z COORDINATES
     X-STRESS           Y-STRESS          Z-STRESS          XY-STRESS         YZ-STRESS        XZ STRESS
       1                 2    4    8    6                0.500000        0.500000       -1.000000
      -0.003596          0.001742          0.001742         -0.000000          0.000000        -0.000000
   -5834.408651         -0.031494         -0.032471         -0.002012          0.001264        -0.000628
       1                 1    3    7    5                0.500000        0.500000       -0.
      -0.003596          0.001742          0.001742          0.000000          0.000000        -0.000000
   -5834.423828         -0.054688         -0.039795          0.001860          0.000342        -0.000652
       1                 2    4    1    3                0.500000        1.000000       -0.500000
      -0.003596          0.001742          0.001742         -0.000000          0.000000        -0.000000
   -5834.411865         -0.039795         -0.030029         -0.000360          0.001225        -0.005466
       1                 5    8    6    7                0.500000       -0.             -0.500000
      -0.003596          0.001742          0.001742         -0.000000          0.000000         0.000000
   -5834.418701         -0.044189         -0.040283         -0.000098          0.000362         0.004191
       1                 1    2    5    6                0.              0.500000       -0.500000
      -0.003596          0.001742          0.001742         -0.000000          0.000000        -0.000000
   -5834.416260         -0.040039         -0.030762         -0.000086          0.003714        -0.001129
       1                 8    4    7    3                1.000000        0.500000       -0.500000
      -0.003596          0.001742          0.001742         -0.000000         -0.000000        -0.000000
   -5834.416992         -0.046143         -0.041504         -0.000088         -0.002076        -0.000159

AVERAGE STRESS FOR ELEMENT  1
   -5834.417480         -0.043945         -0.037354         -0.000071          0.000787        -0.000640
```

YIELD CHECK AFTER 2 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT PCISSON
1	0.003596	0.003266	5834.4	5600.0	1622432.8	0.4844

ELEMENT NUMBER	EQUIVALENT TOTAL STRAIN (PERCENT)	PLASTIC STRAIN COMPONENTS (PERCENT) X-DIR	Y-DIR	Z-DIR
1	0.35586	-0.16294	0.08147	0.08147

01 EXIT IN RETSCP

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UNUSED CORE                             6C455 THRU  64673

  PEGIN EXECUTION.
  1   8   1   8   1   3   0   2   3   C   1
  1           C.              1.0000        -0.
  2           C.              1.0000        -1.0000
  3           1.C000          1.0000         0.
  4           1.0000          1.0000        -1.0000
  5           0.             -0.             0.
  6           C.             -0.            -1.0000
  7           1.C000         -0.             0.
  8           1.C000         -0.            -1.0000
  1   1   1   8
  1        56CC.0000          0.             4.0500           -0.
  1   176500CC.0000           C.3300         9.8000
    1      -0.16294394    C.08147192     0.08147204
  2   4   8   6   1   3   7   5   1   -200.000
  1   0   1   0     -0.             -0.             -0.             -0.
  2   0   1   1     -0.             -0.             -0.             -0.
  3   0   1   1     -C.             -0.             -0.             -0.
  4   0   1   1     -C.             -0.             -0.             -0.
  5   0   0   0     -0.             -0.             -0.             -0.
  6   0   1   1     -C.             -0.             -0.             -0.
  7   0   0   1     -0.             -C.             -0.             -0.
  8   0   1   1     -C.             -0.             -0.             -0.
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YOR644E PRICE
ROTATED SYSTEM

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NCDE	X-CISPL	Y-DISPL	Z-DISPL	X-FCRCE	Y-FORCE	Z-FCRCE
1	0.	-C.2330E-C2	-C.	-J.1584E+05	0.3906E-02	0.7324E-C3
2	-0.	-0.2330E-02	0.233CE-02	-J.1584E+05	0.2197E-02	-0.3418E-02
3	-0.	-C.2330E-02	C.1746E-C8	C.1584E+05	0.3174E-02	0.4395E-02
4	-0.	-C.2330E-02	0.233CE-02	J.1584E+05	0.1709E-02	-C.2686E-02
5	-0.	-0.	-0.	-J.1584E+05	-0.4883E-02	0.415CE-02
6	-0.	0.2095E-08	0.233CE-02	-J.1584E+05	-0.1953E-02	-0.2686E-02
7	-0.	-C.	-C.1281E-C8	0.1584E+05	-0.4639E-02	0.1953E-C2
8	-0.	0.2445E-08	C.233CE-C2	J.1584E+05	-0.1465E-02	-0.1465E-02

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	0.	-0.23297944E-02	-0.
2	-0.	-0.23297940E-02	0.23297951E-02
3	-0.	-0.23297951E-02	0.17462298E-08
4	-0.	-0.23297938E-02	0.23297952E-02
5	-0.	0.	-0.
6	-0.	0.20954758E-08	0.23297936E-02
7	-0.	0.	-0.12805685E-08
8	-0.	0.24447218E-08	0.23297938E-02

```
ELEMENT NUMBER          FACE NODE NUMBERS          X,Y AND Z COORDINATES
    X-STRESS          Y-STRESS          Z-STRESS          XY-STRESS          YZ-STRESS          XZ-STRESS
       1              2     4     8     6             0.500000     0.500000     -1.000000
       0.003589      -0.001185        -0.001185         0.000000        -0.000000          0.000000
   63353.609863       0.001587         0.011963         0.001931        -0.000338          0.001207
       1              1     3     7     5             0.500000     0.500000     -0.
       0.003589      -0.001185        -0.001185        -0.000000         0.000000          0.000000
   63353.621582       0.031128         0.017578        -0.002313         0.000435          0.001255
       1              2     4     1     3             0.500000     1.000000     -0.500000
       0.003589      -0.001185        -0.001185        -0.000000        -0.000000          0.000000
   63353.614258       0.013794         0.014404        -0.000288        -0.000483          0.006083
       1              5     8     6     7             0.500000    -0.           -0.500000
       0.003589      -0.001185        -0.001185        -0.000000         0.000000         -0.000000
   63353.616699       0.017700         0.015137        -0.000014         0.000193         -0.002669
       1              1     2     5     6             0.           0.500000     -0.500000
       0.003589      -0.001185        -0.001185        -0.000000        -0.000000          0.000000
   63353.616211       0.019531         0.015625        -0.000352        -0.002993          0.001593
       1              8     4     7     3             1.000000     0.500000     -0.500000
       0.003589      -0.001185        -0.001185         0.000000         0.000000          0.000000
   63353.614258       0.012329         0.013672         0.000002         0.002764          0.000676

AVERAGE STRESS FOR ELEMENT  1
   63353.616211       0.016968         0.015869        -0.000248        -0.000048          0.001207
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YIELD CHECK AFTER 1 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.003589	0.003259	5833.9	5600.0	1625296.3	0.4843

ROTATED SYSTEM

NODE	X-DISPL	Y-DISPL	Z-DISPL	X-FORCE	Y-FORCE	Z-FORCE
1	-0.	-0.2884E-02	0.	-0.1458E+04	-0.7813E-02	-0.1099E-01
2	0.	-0.2884E-02	0.2884E-02	-0.1458E+04	-0.9521E-02	0.7568E-02
3	0.	-0.2884E-02	0.8382E-08	0.1458E+04	-0.9766E-02	-0.9766E-02
4	0.	-0.2884E-02	0.2884E-02	0.1458E+04	-0.9766E-02	0.1099E-01
5	0.	0.	0.	-0.1458E+04	0.6836E-02	-0.6836E-02
6	0.	0.2328E-07	0.2884E-02	-0.1458E+04	0.1196E-01	0.1074E-01
7	0.	0.	-0.1397E-07	0.1458E+04	0.8789E-02	-0.9521E-02
8	0.	0.2235E-07	0.2884E-02	0.1458E+04	0.9521E-02	0.8789E-02

NODE	X-DISPLACEMENTS	Y-DISPLACEMENTS	Z-DISPLACEMENTS
1	-0.	-0.28838025E-02	C.
2	0.	-0.28837929E-02	0.28838106E-02
3	0.	-0.28838073E-02	C.83819032E-08
4	0.	-0.28837929E-02	C.28838236E-02
5	0.	-0.	C.
6	0.	0.23283064E-07	C.28838064E-02
7	0.	-0.	-C.13969839E-07
8	0.	0.22351742E-07	0.28837956E-02

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ELEMENT NUMBER          FACE NODE NUMBERS          X,Y AND Z COORDINATES
   X-STRESS             Y-STRESS          Z-STRESS         XY-STRESS        YZ-STRESS        XZ-STRESS
       1                2    4    8    6              0.500000      0.500000      -1.000000
       0.003585        -0.001739         -0.001739        -0.000000        -0.000000        -0.000000
    5833.864590        -0.045838         -0.036377        -0.000263        -0.002510        -0.000458
       1                1    3    7    5              0.500000      0.500000      -0.
       0.003585        -0.001739         -0.001739        -0.000000        -0.000000        -0.000000
    5833.882324        -0.016602         -0.029297        -0.001322        -0.001569        -0.000438
       1                2    4    1    3              0.500000      1.000000      -0.500000
       0.003589        -0.001739         -0.001739        -0.000000        -0.000000         0.000000
    5833.866652        -0.033936         -0.040527        -0.000800        -0.002502         0.005856
       1                5    8    6    7              0.500000     -0.            -0.500000
       0.003585        -0.001739         -0.001739        -0.000000        -0.000000        -0.000000
    5833.877686        -0.029785         -0.026123        -0.000778        -0.001565        -0.006748
       1                1    2    5    6              0.            0.500000      -0.500000
       0.003585        -0.001739         -0.001739        -0.000000        -0.000000         0.000000
    5833.874269        -0.026367         -0.026855        -0.000798        -0.007808         0.000044
       1                3    4    7    3              1.000000      0.500000      -0.500000
       0.003585        -0.001739         -0.001739        -0.000000         0.000000        -0.000000
    5833.874023        -0.034668         -0.037354        -0.000781         0.003728        -0.000920

AVERAGE STRESS FOR ELEMENT  1
    5833.874512        -0.030273         -0.031982        -0.000793        -0.002032        -0.000442
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YIELD CHECK AFTER 2 ITTERATIONS

ELEMENT	TOTAL STRAIN	PLASTIC STRAIN	EFFECTIVE STRESS	YIELD STRESS	SECANT MODULUS	SECANT POISSON
1	0.003589	0.003259	5833.9	5600.0	1625295.5	0.4843

ELEMENT NUMBER	EQUIVALENT TOTAL STRAIN (PERCENT)	PLASTIC STRAIN COMPONENTS (PERCENT) X-DIR	Y-DIR	Z-DIR
1	C.35520	C.16295	-0.08147	-0.08147

*0. EXIT IN RETSCP

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